

# Quantum Computation and Quantum Information

A project report  
submitted in partial fulfillment for the award of the degree of  
Master of Science in Physics

by

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## DECLARATION

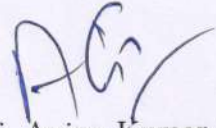
I hereby declare that the project work entitled "Quantum Computing and Quantum Information" is a work carried out by me under the supervision of Prof. Anjan Kumar Giri, Indian Institute Of Technology, Hyderabad.

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## CERTIFICATE

This is to certify that the project work titled "Quantum Computation and Quantum Information" submitted to the Indian Institute Of Technology, Hyderabad, in partial fulfillment of requirements for the award of the degree of Masters in Science in Physics, is a work done by Tony Narzary, during the period of his study, Aug 2018 - April 2019, in the Indian Institute Of Technology, Hyderabad, under my supervision and guidance. This thesis has not been submitted for the award of any other Degree/Diploma/Fellowship or any other similar title.



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## **ABSTRACT**

In this project, basics of quantum computation and quantum information is studied. The present work starts with the study of linear algebra which is essential to describe the state quantum systems. A basic description of Church Turing thesis is discussed. The basics of linear algebra is used to describe qubits and gates. Finally using circuit model of computation, circuits are used to implement quantum algorithms.

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# Chapter 1

## Background

### 1.1 Introduction and aim of the project

Computers are designed to solve problems through algorithms. Computers store and process data. The storage and processing of data involves physical processes. Therefore, it becomes important to understand the physical quantum processes like superposition, entanglement etc.. Linear Algebra is the mathematical language to describe quantum systems. The aim of this project is to learn the basics of linear algebra and circuit model of computing. Using this tools, to study the implementation of quantum circuits of algorithms.

### 1.2 Linear Algebra and Dirac Notation

A quantum state is represented by a vector in Hilbert Space. A vector 'a' is represented in Dirac notation by  $|a\rangle$ . The dual vector of 'a' is represented by  $\langle a|$  in dual vector space. The vector spaces that we will consider are defined over complex numbers and are finite-dimensional. The inner product is represented by  $\langle a|a\rangle$ .

We will often measure quantum states in computational basis. For a quantum system with two states, we can represent the state of the system using  $|0\rangle$  and  $|1\rangle$ . Due to superposition of states the state can be mathematically represent as

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$$

where  $\alpha$  and  $\beta$  are complex numbers. The square of the coefficient of each state is the probability of measuring the corresponding state. This gives rise to the condition

$$|\alpha|^2 + |\beta|^2 = 1$$

Each basis vector can be associated with an corresponding column matrix.

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Quantum systems interacting with each other can be represented using tensor product (Note: Entangled quantum systems cannot be represented using tensor product.)

Two quantum systems with two states can be represented by the tensor product of two individual qubit state,  $\otimes$ . Let

$$|\psi_1\rangle = \alpha_1 |0\rangle + \beta_1 |1\rangle$$

$$|\psi_2\rangle = \alpha_2 |0\rangle + \beta_2 |1\rangle$$

The joint state is

$$|\psi_1\psi_2\rangle = \alpha_1\alpha_2 |00\rangle + \alpha_1\beta_2 |01\rangle + \beta_1\alpha_2 |10\rangle + \beta_1\beta_2 |11\rangle$$

Equivalently in matrix representation

$$|\psi_1\psi_2\rangle = \alpha_1\alpha_2 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \alpha_1\beta_2 \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \beta_1\alpha_2 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \beta_1\beta_2 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

### 1.2.1 Inner Product

For a hilbert space,  $H$  there exist a hilbert space  $H^*$  such that the vector of  $H^*$  (also known as dual vector), represented as  $\langle a|$  has the action,

$$\langle a| : |b\rangle \longrightarrow \langle a|b\rangle \in C$$

Inner product between two vectors  $|u\rangle$  and  $|v\rangle$  is represented by  $\langle u, v\rangle$  or  $\langle u|v\rangle$ . An inner product is a function having the following properties:

1. Linearity:  $\langle u|\sum_{i=1} \lambda_i v_i\rangle = \sum_{i=1} \lambda_i \langle u|v_i\rangle$
2. Conjugate-Commutative:  $\langle u|v\rangle = \langle v|u\rangle^*$
3. Non Negativity:  $\langle u|v\rangle \geq 0$



In terms of matrix representation, the dual vector is represented by a row matrix and the vector by a column matrix. The inner product is the usual matrix multiplication.

Two vectors are called orthogonal if their inner product is 0. A vector is normal if its inner product is 1. A set of unit vectors that are mutually orthogonal is called an orthonormal set. If all vectors in a space can be written in terms of a set of orthonormal vectors, the orthonormal set is called orthonormal basis.

$$|\psi\rangle = \sum a_n |b_n\rangle, \quad a_n \in C, |b_n\rangle \in H$$

$$\langle b_m | b_n \rangle = \delta_{mn} \quad \delta_{mn} = \begin{cases} 1, & \text{if } m = n, \\ 0, & \text{if } m \neq n. \end{cases}$$

## 1.2.2 Operator

### Linear Operator

A linear operator on a vector space  $H$  is a linear transformation  $T : H \rightarrow H$  of the vector space to itself (i.e. it is a linear transformation which maps vectors in  $H$  to vectors in  $H$ ).

### Adjoint Operator

Suppose  $T$  is an operator on  $H$ . Then the adjoint of  $T$ , denoted  $T^\dagger$ , is defined as that linear operator on  $H$  that satisfies

$$(\langle \psi | T^\dagger | \phi \rangle)^* = \langle \phi | T | \psi \rangle, \quad \forall | \phi \rangle, | \psi \rangle \in H$$

In matrix representation,  $T^\dagger$  is the complex conjugate transpose of  $T$ .

### Unitary Operator

An operator  $U$  is called unitary if  $U^\dagger = U^{-1}$ , where  $U^{-1}$  is the inverse of  $U$ .

$$UU^\dagger = I$$

### Eigen values

A vector  $\psi$  is called an eigenvector of an operator  $T$  if

$$T | \psi \rangle = c | \psi \rangle$$

for some constant  $c$ . The constant  $c$  is called the eigenvalue of  $T$  corresponding to the eigenvector  $| \psi \rangle$ .

## Hermitian Operator

An operator  $T$  in a Hilbert space  $H$  is called Hermitian (or self-adjoint) if

$$T^\dagger = T$$

The eigen values of Hermitian Operators are real. All quantum operators corresponding to physical variables are Hermitian, since the physical quantities measured are real.

## 1.3 Introduction to Quantum Physics

□ In quantum physics, an experiment behaves as if it proceeds down all possible paths simultaneously. A complex probability amplitude is associated with each path. The sum of all the squares of the complex amplitudes associated with the path leading to a particular outcome is the probability of that particular outcome. A quantum computer behaves in much the same way. □ The computation proceeds down all possible paths at once, and each path has associated with it a complex amplitude. To determine the probability of any final state of the machine, we add the amplitudes of all the paths which reach that final state, and then square the absolute value of this sum.

## 1.4 Introduction to Church Turing Thesis

Physical computers are used to perform algorithms. Computers require resources, time and space, to solve problem. Space refers to the amount of memory required by the computer to compute. The time required by computers are function of the input bits. For example for input bits of  $n$ , time required could be  $f(n) = n^2 + 1$ . A computer with different architecture may required different time,  $f(n) = 3n^3 + 2n^2 + 5$ . We can use upper bound time represented as  $O(n^2)$  and  $O(n^3)$  for the former and the latter computer respectively to make things easier.

There are computers with different architecture so the study of computability and computational complexity theory may be wrongly thought to be done for each individual computer.

### 1.4.1 Church-Turing thesis

The class of functions computable by a Turing machine corresponds exactly to the class of functions which we would naturally regard as being computable by any algorithm [2].

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \left( \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)$$

### 1.4.2 Turing's Machine

Turing's Machine consists of a program, finite state control, read and write head, and tape. finite state control. The finite state control consists of states  $q_1, q_2, \dots, q_n$  with initial state,  $q_s$  and halting state,  $q_h$ . Tape is a linear object extending to infinity in one direction. Tape consists of square which acts like a memory. The head is used either to read or write on the tape.

The programs are written as such:

$$\langle q_s, x, q_1, x_1 \rangle$$

The program starts with  $q_s$  state, the head reads the data  $x$ , the state is changed to  $q_1$ , the head rewrites the data as  $x_1$  and shifts the head towards to next square in the positive direction. The Turing Machine is capable of computing functions from non-negative inputs to non-negative outputs. Turing machine can simulate any other Turing machines.

A computer is generally considered to be a universal computational device; i.e., it is believed able to simulate any physical computational device with a cost in computation time of at most a polynomial factor [1].

# Chapter 2

## Quantum Computation

### 2.1 Qubits

#### 2.1.1 Single Qubits

Qubits are the smallest data unit. The state of the qubits can be represented by unit vectors in Hilbert space. Qubits are quantum mechanical system that are observed in two states:  $|0\rangle$  and  $|1\rangle$ . In a quantum computer the state of each bit is permitted to be any quantum-mechanical state of a qubit, or two-level quantum system. Computation proceeds by a succession of “two-qubit quantum gates” [3]. It can be expressed as a linear combination of both the states:

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$$

where  $\alpha$  and  $\beta$  are complex amplitudes of  $|0\rangle$  and  $|1\rangle$  respectively.  $|\alpha|^2$  and  $|\beta|^2$  represent the probability of observation of  $|0\rangle$  and  $|1\rangle$  on measurement respectively. Therefore,

$$|\alpha|^2 + |\beta|^2 = 1$$

Equivalently qubits can also be represented using matrices:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Qubits can be physically realized by:

1. Photons passing through two distinct paths.
2. Spin of a particle.
3. Presence or absence of a particle.

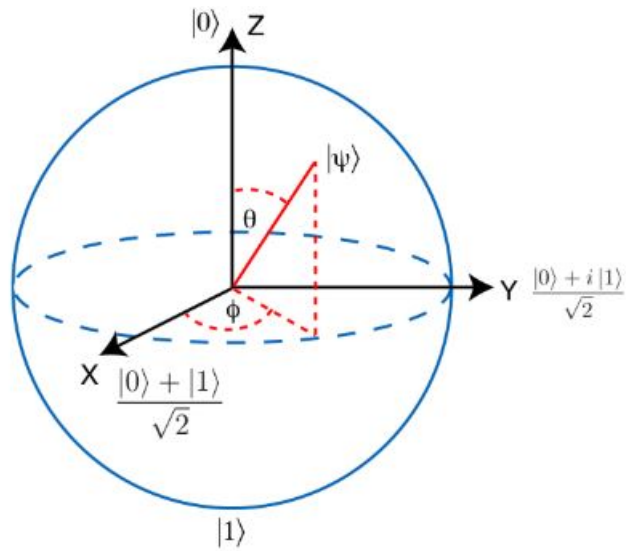


Figure 2.1: Bloch Sphere [2]

### Bloch Representation of single qubits

It is a geometrical representation of qubits in Bloch sphere. Bloch sphere has a radius of one unit. Qubits can be represented as

$$|\psi\rangle = \cos\frac{\theta}{2} |0\rangle + e^{i\phi} \sin\frac{\theta}{2}$$

where  $\theta$  and  $\phi$  are real numbers.  $\phi$  is the phase difference between  $|0\rangle$  and  $|1\rangle$ . The positive Z direction represents  $|0\rangle$  state and the negative Z direction represents the  $|1\rangle$  state. The superposition states lie between these two poles.

### 2.1.2 Multiple Qubits

A qubit is not of practical use. There is a need for more qubits to process real world data. Two qubits can be represented by the tensor product of two individual qubit state,  $\otimes$ .

### 2.1.3 Time Evolution of Qubits

Quantum systems change over time, so the quantum state must be describe by function in time,  $|\psi(t)\rangle$ . Mathematically a linear operator is used to describe a change in quantum state.

$$U \sum a_i \psi_i = \sum a_i U \psi_i$$

The state's evolution in the course of time t is described by a unitary operator U on this vector space, i.e., a linear transformation that is bijective and length

preserving. This unitary evolution on a normalized state vector is known to be the correct physical description of an isolated system evolving in time according to the laws of quantum mechanics[4].

$$|\psi\rangle \longrightarrow U |\psi\rangle$$

$$\langle\psi|\psi\rangle = 1$$

$$\langle\psi|U^\dagger U|\psi\rangle = 1$$

$$U^\dagger U = 1$$

### 2.1.4 Measurement

To measure a quantum system, it must interact with the outer environment. A quantum state in

$$|\psi\rangle = \sum a_i |\phi_i\rangle$$

where  $|\phi_i\rangle$  forms an orthonormal basis and  $a_i \in \mathbb{C}$ , when measured in orthonormal basis leaves the system in state  $|\phi_i\rangle$  with probability  $|a_i|^2$ .

## 2.2 Quantum Gates

The state of the qubits need to be altered as per desired to give meaningful output. The basic blocks that change the state of a qubit is called Quantum Gate. Quantum Gates are represented mathematically as unitary matrices applied on the state of the qubit.

$$UU^\dagger = I$$

where  $U^\dagger$  is the adjoint of U and I is the identity matrix.

### 2.2.1 One Qubit Gate

Some of the gates that are commonly used are:

#### Hadamard Gate

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Effect of Hadamard Gate on  $\psi = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$$H\psi = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) + \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$$H\psi = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$$

where  $|+\rangle = |0\rangle + |1\rangle$  and  $|-\rangle = |0\rangle - |1\rangle$

Therefore the effect of Hadamard Gate is to convert  $|0\rangle$  to  $|+\rangle$  and  $|1\rangle$  to  $|-\rangle$ . In the Bloch sphere the application of Hadamard Gate on  $|0\rangle$  amounts to rotation of the qubit  $90^\circ$  about Y-axis.

### Identity Matrix

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

### X Gate

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\alpha |0\rangle + \beta |1\rangle \xrightarrow{X} \beta |0\rangle + \alpha |1\rangle$$

The effect of X on the qubit is to swap the coefficient of the states. Initially, if the qubit is in  $|0\rangle$  state, X will convert it into  $|1\rangle$ .

### Y Gate

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

### Z-Gate

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

X-Gate, Y-Gate and Z-Gate corresponds to gates that rotate the qubit about x-axis, y-axis and z-axis respectively. These gates along with the identity gate are called Pauli Gates. Pauli gates are important for quantum computing is that they span the vector space formed by all 1-qubit operators. In particular, this means that any 1-qubit unitary operator can be expressed as a linear combination of the Pauli gates.

### Rotation Gates

$$R_x(\theta) = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) & -i\sin\left(\frac{\theta}{2}\right) \\ -i\sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{pmatrix}$$

$$R_y(\theta) = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) & -\sin\left(\frac{\theta}{2}\right) \\ \sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{pmatrix}$$

$$R_z(\theta) = \begin{pmatrix} e^{-i\left(\frac{\theta}{2}\right)} & 0 \\ 0 & e^{i\left(\frac{\theta}{2}\right)} \end{pmatrix}$$

## Phase Shift Gates

**T-Gate:**

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

If we have an initial state of  $|+\rangle$ , the action of T on  $|+\rangle$  is to rotate the  $\frac{\pi}{4}$  about Z-axis in the bloch sphere.  $T^\dagger$  rotates  $|+\rangle$ ,  $-\frac{\pi}{4}$  about Z-axis in the bloch sphere.

**S-Gate**

$$S = T^2$$

$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

S is application of T-gate twice hence  $|+\rangle$  rotates  $\frac{\pi}{2}$  about Z-axis in the bloch sphere.  $S^\dagger$  rotates  $|+\rangle$ ,  $-\frac{\pi}{2}$  about Z-axis in the bloch sphere.

## 2.2.2 Multiple Qubit Gate

**C-NOT gate**

C-NOT gate has two input qubits.  $|A\rangle$  is the control qubit and  $|B\rangle$  is the target qubit. The control qubit is unchanged after the application of C-NOT gate. The target qubit is transformed to  $|A \oplus B\rangle$ , where  $\oplus$  is addition modulo two. The matrix



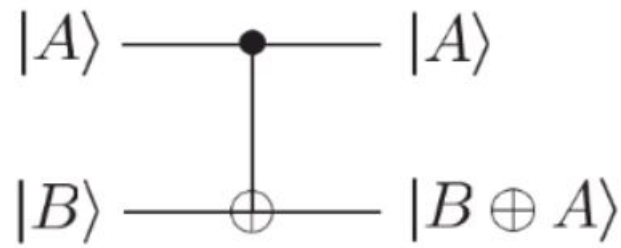


Figure 2.2: C-NOT gate

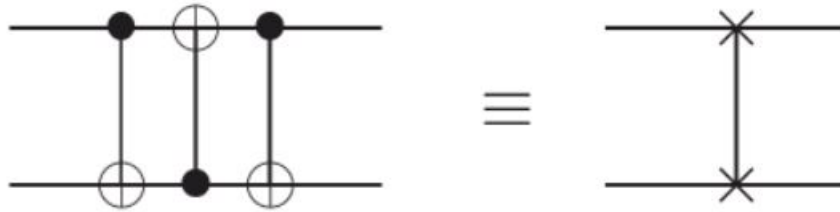


Figure 2.3: Circuit to swap two qubits

representation of C-NOT gate is

$$C_X = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

C-NOT can be used to swap two qubits. Mathematically,

$$\begin{aligned} |a, b\rangle &\longrightarrow |a, a \oplus b\rangle \\ &\longrightarrow |a \oplus a \oplus b, a \oplus b\rangle \\ &\longrightarrow |b, a \oplus b\rangle \\ &\longrightarrow |b, a \oplus b \oplus b\rangle \\ |a, b\rangle &\longrightarrow |b, a\rangle \end{aligned}$$

### Toffoli Gate

Toffoli Gate is an extension of C-NOT gate. It has two control qubits and one target qubit.

### Controlled-U Gate

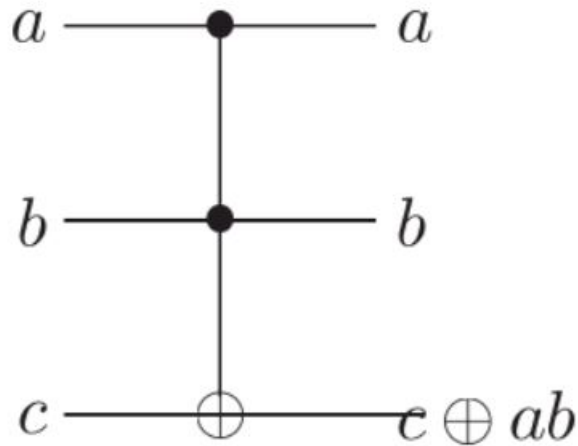


Figure 2.4: Toffoli gate

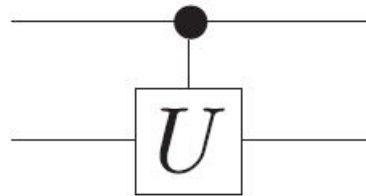


Figure 2.5: c-U Gate

A controlled-U gate, c-U for a single qubit gate  $U$  is a two qubit gate defined as

$$c-U |0\rangle |\psi\rangle = |0\rangle |\psi\rangle$$

$$c-U |1\rangle |\psi\rangle = |1\rangle U |\psi\rangle$$

## 2.3 Quantum Circuit Model

A circuit model is a network of gates used to manipulate the input qubits. Input qubits are fed to quantum gates using quantum channels. Quantum channels are represented by horizontal lines and quantum gates are represented by blocks. The circuit is divided into time slices, the application of one gate requires a time slice. The qubits propagate from left to right with the passage of time. In practice the input of a quantum circuit is usually initialized to  $|0\rangle$ . A quantum circuit can be characterized by the number of time slices, depth and the number of wires, space. The initial state can be represented by  $|\psi_i\rangle = |0\rangle |0\rangle |0\rangle |0\rangle$ . Measurement of the final

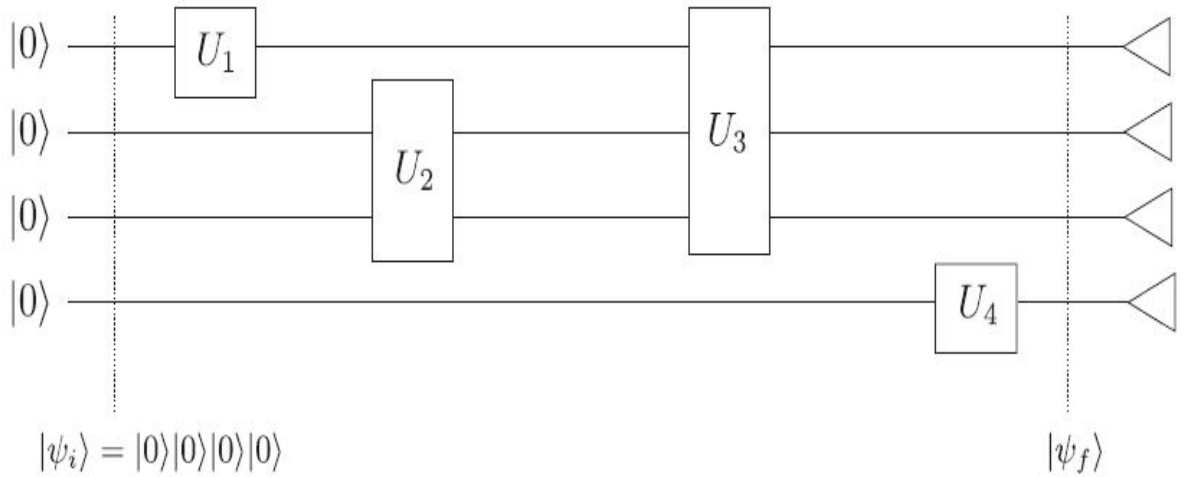


Figure 2.6: A Quantum Circuit [5]

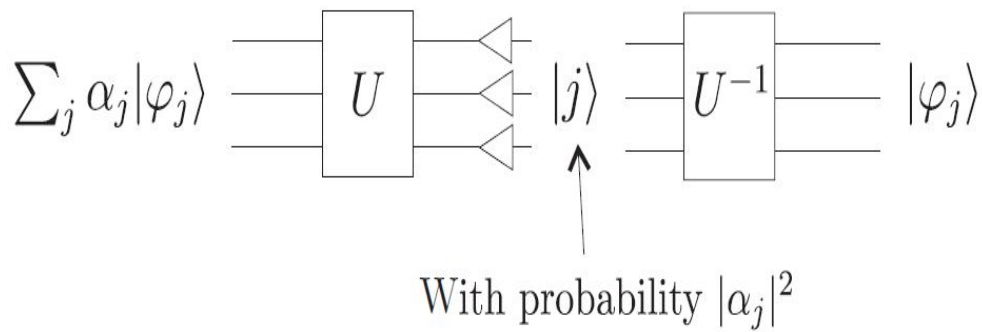


Figure 2.7: Approach 1: Circuit implementing measurement [5]

state of the qubit in the computational basis is represented by a triangle in the circuit.

**Theorem:** Suppose  $U$  is a 1-qubit unitary gate. Then there exist real numbers  $\alpha, \beta, \gamma,$  and  $\delta$  such that

$$U = e^{i\alpha} R_z(\beta) R_y(\gamma) R_z(\delta)$$

### 2.3.1 Implementing Measurements with Quantum Circuits

**Approach 1:** Let us consider a quantum state,

$$|\psi\rangle = \sum a_i \phi_i$$

where  $\phi_i$  forms an orthogonal basis. A Unitary Gate,  $U$  is constructed such that,

$$U |\phi_i\rangle = |i\rangle$$

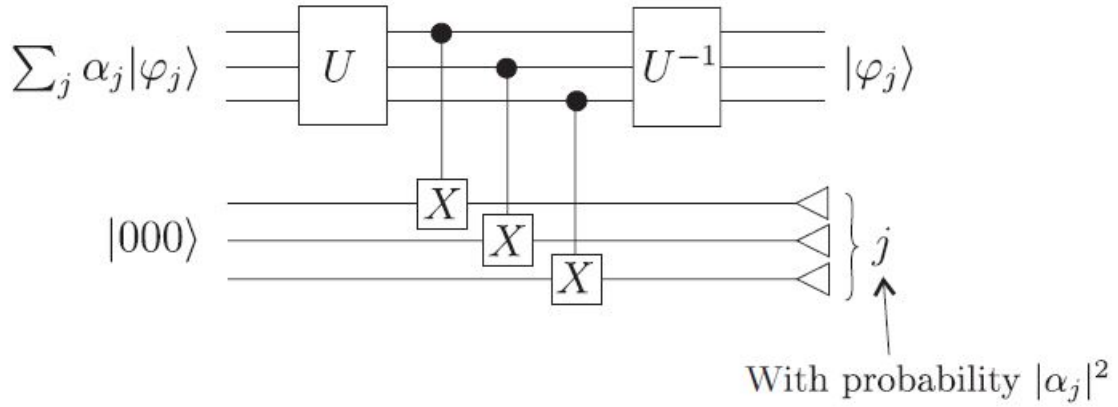


Figure 2.8: Approach 2: Circuit implementing measurement [5]

where  $|i\rangle$  is the corresponding computational basis state. Finally  $U^{-1}$  is applied to change it back to the orthogonal basis. The measurement results in the state  $|i\rangle$  with probability  $|\alpha_i|^2$ .

**Approach 2:** In an alternative approach instead of directly measuring after the application of the unitary operator, we can copy the qubits to the ancillary register and measure it from the ancillary register.

## 2.4 Quantum Algorithms

The two quantum mechanical effects that quantum algorithms can exploit to outperform classical algorithms are superposition of states and entanglement [6]. Grover's Algorithm is implemented on ibmqx4. ibmqx4 is a 5-qubit IBM quantum computer. In real computers, there is no full connectivity among the qubits, hence two qubit gates can only be applied between qubits with connectivity. In the ibmqx4, which has 5 qubits, there are 6 connections, i.e., there are only 6 pairs of qubits to which a CNOT gate can be applied.

### 2.4.1 Grover's Algorithm

**Objective:** To search for a specific item within a randomly ordered database of  $N$  items with probability  $> \frac{1}{2}$ .

**Background:**

**Oracle:** Let  $X = x_1, x_2, \dots, x_n$  be a set of  $N$  items and  $x^*$  is the item that has to be

found. We define a function,

$$f(x) = \begin{cases} 1, & x = x^* \\ 0, & x \neq x^* \end{cases}$$

The action of the oracle operator is

$$|x\rangle |q\rangle \xrightarrow{O} |x\rangle |q \oplus f(x)\rangle$$

where  $|x\rangle$  is one of the items from the database and  $q$  is a target qubit.

In practice  $|q\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$  Therefore the action of the oracle is:

$$|x\rangle \left| \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \right\rangle \xrightarrow{O} (-1)^{f(x)} |x\rangle \left| \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \right\rangle$$

A phase factor of  $\pi$  is multiplied to  $|x\rangle$  when the solution is found or else the qubits remain unchanged.

### Grover's Operator [\[2\]](#)

$$G = (2|\psi\rangle\langle\psi| - I)O$$

where  $\psi$  is the uniform superposition of all states and  $O$  is the oracle.

The action of  $2|\psi\rangle\langle\psi| - I$  on the qubit is to invert the coefficient of states about the average of the coefficient of all states.

The total action of Grover's operator is to increase the probability(coefficient) of the solution.

#### Algorithm:

- Apply Oracle
- Apply Grover's operator
- Iterate for  $O(\sqrt{N})$
- Measure  $|x\rangle$

## 2.5 Implementation of Grover's Algorithm on ibmqx4

Grover's Algorithm was implemented to search a solution for

$$f(x_1, x_2) = x_1 x_2 = 1.$$

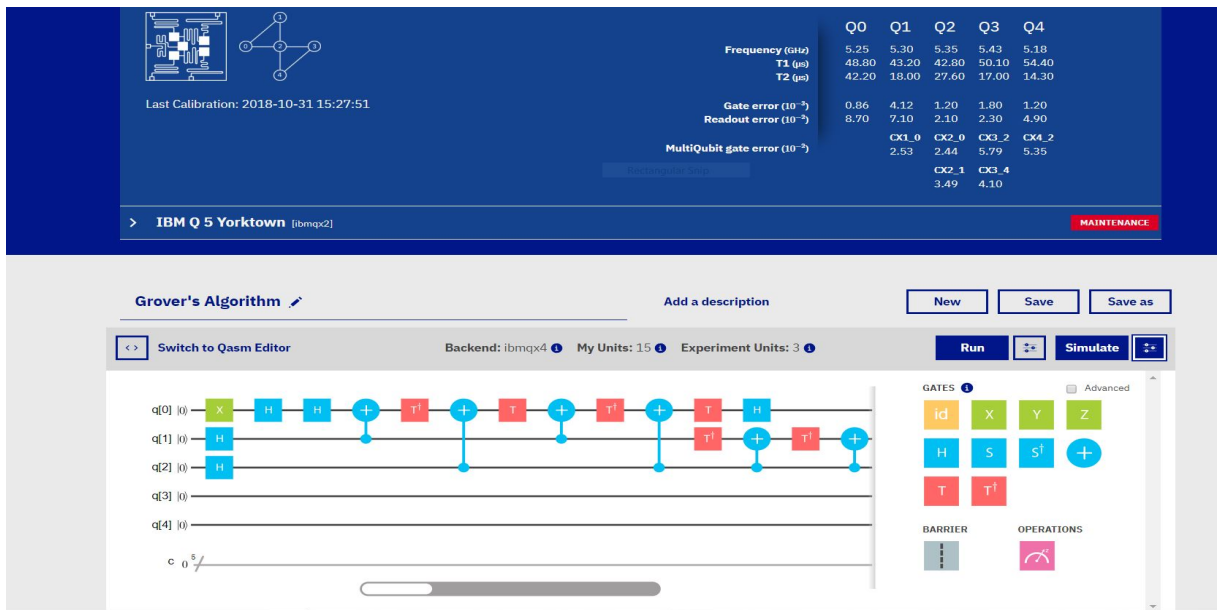


Figure 2.9: IBM window [7]

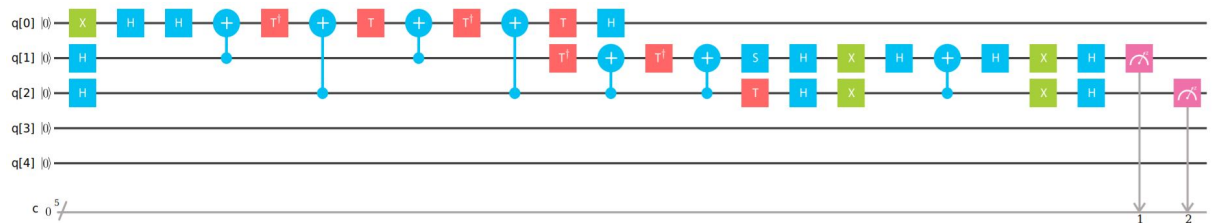


Figure 2.10: Grover's Algorithm Circuit

## Grover's Algorithm

Device: ibmqx4

Quantum State: Computation Basis

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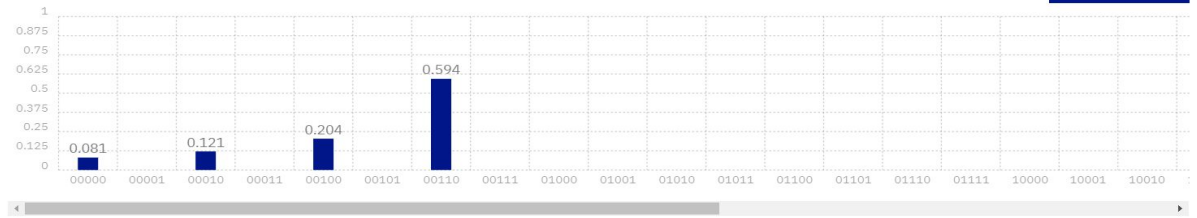


Figure 2.11: Grover's Algorithm Results

$q[0]$  is chosen as the ancillary bit.  $q[1]$  and  $q[2]$  are the register bits. The probability of finding the correct answer (1,1) was 59.4%.

# Chapter 3

## Quantum Information

Here the protocols to share quantum information through quantum channels is discussed. Let us consider two parties 'Alice' and 'Bob' communicating through the quantum channel. The protocols require that both the parties share a entangled pair of qubits is Bell state.

$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

### 3.1 Super Dense Coding

**Objective:** To send classical bits through quantum channel.

The basic idea of quantum dense coding is that quantum mechanics allows one to encode information in the quantum states that is denser than classical coding[8].

Alice and Bob share an entangled state. Using the Pauli Gates, depending on the bits to be send, Pauli Gates are operated on the entangled state to alter its

To send	Transformation
00	$I \otimes I : \frac{1}{\sqrt{2}}( 00\rangle +  11\rangle) \mapsto \frac{1}{\sqrt{2}}( 00\rangle +  11\rangle) =  \beta_{00}\rangle$
01	$X \otimes I : \frac{1}{\sqrt{2}}( 00\rangle +  11\rangle) \mapsto \frac{1}{\sqrt{2}}( 01\rangle +  10\rangle) =  \beta_{01}\rangle$
10	$Z \otimes I : \frac{1}{\sqrt{2}}( 00\rangle +  11\rangle) \mapsto \frac{1}{\sqrt{2}}( 00\rangle -  11\rangle) =  \beta_{10}\rangle$
11	$Z \cdot X \otimes I : \frac{1}{\sqrt{2}}( 00\rangle +  11\rangle) \mapsto \frac{1}{\sqrt{2}}( 01\rangle -  10\rangle) =  \beta_{11}\rangle$

Figure 3.1: Transformation table [5]



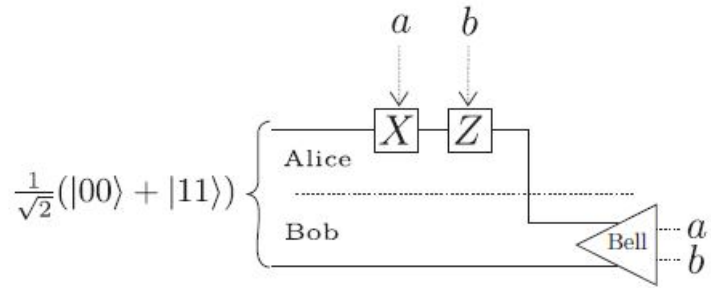


Figure 3.2: Measuring circuit [5]

state. If Alice has to send two bits through the channel, the list of operations to be carried out are shown in the figure 3.1. For each unique combination of bits to sent, the corresponding operation results in an unique bell state. On measurement, Bob would identify the bits by identifying the different bell states.

This has been experimentally demonstrated by Mattle et al. [9].

# Chapter 4

## Results

We studied the basics of linear algebra, basics of quantum physics and Church Turing thesis, the knowledge of which was used to describe qubits, gates and build quantum circuits. Quantum circuits for Grover's algorithms were studied and implemented on ibmqx4. A basic study of information transfer through super dense coding was also studied.

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