Chapter 1

Introduction

1.1 Motivation

The Finite Element Method (FEM) has been found wide used of enormous application in many fields. FEM approximation is interpolatory in nature. This interpolatory nature of approximation in FEM makes the solution mesh dependent. One possibility to achieve accuracy in solution is by adaptive refinement of mesh. However such schemes are computationally expensive. FEM does not suit well to treat problems with discontinuities (i.e. Crack propogation) that do not align with the mesh. Meshless Methods (MMs) have been used with objective of eliminating part of difficulties associated with reliance on mesh to construct the approximation. MMs are based on approximations that are not interplatory in nature but are based on alternative strategies like for instance the moving least square (MLS) approach. The problem associated with these methods is that they are computationally expensive and non interpolatory nature requires Lagrange multipliers for imposition of boundary conditions.

There has been a need for developing better approximation schemes that have interpolatory characteristics, consistency and stability. Also in using such methods for fracture problems, the methods should be capable of reproducing the actual stress/ strain field ahead of the crack tip. To this end, there has been many works on improving the classical FEM approximations and one such methodology was coined as Extended Finite Element Methods (XFEM). In these methods the approximation is enriched by some additional enrichment functions. However there were difficulties associated with the method as regards making appropriate choice on the enrichment functions.

Minimizing the error between approximation solution and exact solution can make appropriate choice on the enrichment functions belonging to a particular function space. Proper Orthogonal Decomposition (POD) is done on the fact that distance between member of ensemble and subspace is minimal. An XFEM with choice on enrichment functions based on POD is termed as Generalized Finite Element Method (GFEM) [10].

In this work POD has been used for finding out the enrichment basis function for GFEM approximation [10]. Such a enrichment function accounts for the oscillatory nature of the solution ahead of singular regions. The global approximation has been constructed by combining the local bases with partition of unity functions (i.e. classical basis function).

1.2 Outline of Thesis

The thesis has been divided into the 9 chapters. The first chapter contains the motivation and second chapter contains the literature survey of XFEM/GFEM. Third chapter contains the review of fracture mechanics. Realization of the capability of GFEM has been illustrated through the shape functions in 1-D and 2-D in the following chapter. Different examples of enrichment functions have been demonstrated in this chapter. Two scales GFEM has two different orders of the approximation, which results into the inter-boundary conflict for the neighboring elements to enriched elements. The blending element concept has been introduced to ensure the continuity among the two different approximations. Next chapter continues the discussion about the blending element. Fourth chapter contains the mathematical foundation of Galerkin formulation of the GFEM and the numerical integration schemes respectively. This chapter also contains the discussion over the implementation issues of the GFEM. Stress analysis of the fracture problems has been shown in the seventh chapter. Eighth chapter contains the sensitivity analysis of the edge crack problem. Final chapter ends with a conclusion and the Scope of the future work.