Packet Size Optimization for Multiple Input Multiple Output Cognitive Radio Sensor Networks aided Internet of Things.

Chitradeep Majumdar*, Doohwan Lee†, Aaqib Ashfaq Patel*, S. N. Merchant*, U. B. Desai ‡
+iIT Bombay, † RCAST, The University of Tokyo, ‡ IIT Hyderabad
email: cm6v07@ee.iitb.ac.in, leedh@mlab.t.u-tokyo.ac.jp, aaqib@ee.iitb.ac.in, merchant@ee.iitb.ac.in, ubdesai@iith.ac.in

Abstract—The determination of Optimal Packet Size (OPS) for Cognitive Radio assisted Sensor Networks (CRSNs) architecture is non-trivial. State of the art in this area describes various complex techniques to determine OPS for CRSNs. However, it is observed that under high interference from the surrounding users, it is not possible to determine a feasible optimal packet size of data transmission under the simple point-to-point CRSN network topology. This is contributed primarily due to the peak transmit power constraint of the cognitive nodes. To address this specific challenge, this paper proposes a Multiple Input Multiple Output based Cognitive Radio Sensor Networks (MIMO-CRSNs) architecture for futuristic technologies like Internet of Things (IoT) and machine-to-machine (M2M) communications. A joint optimization problem is formulated taking into account network constraints like the overall end to end latency, interference duration caused to the non-cognitive users, average BER and transmit power. We propose our Algorithm-1 based on generic exhaustive search technique blue to solve the optimization problem. Furthermore, a low complexity suboptimal Algorithm-2 based on solving classical Karush-Kuhn-Tucker (KKT) conditions is proposed. These algorithms for MIMO-CRSNs are implemented in conjunction with two different channel access schemes. These channel access schemes are Time Slotted Distributed Cognitive Medium Access Control denoted as MIMO-DTS-CMAC and CSMA/CA assisted Centralized Common Control Channel based Cognitive Medium Access Control denoted as MIMO-CC-CMAC. Simulations reveal that the proposed MIMO based CRSN network outperforms the conventional point-to-point CRSN network in terms of overall energy consumption. Moreover, the proposed Algorithm-1 and Algorithm2 shows perfect match and the implementation complexity of Algorithm-2 is much lesser than Algorithm-1. Algorithm-1 takes almost 680 ms to execute and provides OPS value for a given number of users while Algorithm-2 takes 4 to 5 ms on an average to find the optimal packet size for the proposed MIMO-CRSN framework.

Index Terms—Optimal packet size, cognitive radio sensor networks, energy-efficiency, quadrature amplitude modulation, convex optimization, medium access control.

I. INTRODUCTION

Cognitive Radio (CR) has emerged as one of the most blue sought topic for research in the area of wireless communication over the past few years. Considering the exponential rise in the number of mobile phone users in the last decade, it was imperative to come up with a robust technology which would suffice this ever increasing need. The opportunistic spectrum utilization feature of cognitive radio was envisaged to be a promising solution to deal with this challenge [1]. Over the years as the CR technology evolved, the inherent challenges associated with this technology became more and more evident. Various network paradigms based on CR and its variants have been extensively explored by the researchers [2]. Furthermore, it became clear that CR technology which was primarily aimed for mobile communication can now be extended to other upcoming communication paradigms like Internet of Things (IoT) and Machine-to-Machine communications which are likely to be a part of the future wireless communication standards like LTE-A and 5G [3], [4]. These futuristic paradigms like IoT operates on the fundamental principle of data sensing, data acquisition and reliable data transmission from our physical surroundings to a remote processing unit which would process the received data and provide us with some useful information which improves our daily life. This typically involves large number of sensor nodes that continuously gather raw data from different applications. The pervasiveness, scalability of these nodes in terms of its number and the requisite for seamless reliable data transfer makes an IoT application somewhat distinct from the conventional wireless sensor network. These sensor nodes are often subjected to operational limitations like power constraint, limited coverage area and interference from other services coexisting the same frequency band. This motivated researchers to adapt the CR technology into conventional sensor network architecture to come up with Cognitive Radio based Sensor Networks (CRSNs) which could support the demand for IoT based applications in a more efficient way [5], [6]. In this paper we define cognitive nodes or secondary users as the users which have the cognitive channel sensing and switching capabilities whereas the sensor nodes which are not equipped with this cognitive feature or any other services operating within the same frequency band are coined as non-cognitive users.

This paper proposes a novel technique to determine the optimal packet size for a MIMO based CRSN architecture denoted as MIMO-CRSN. The motivation for data transmission with optimal packet size is clearly established by considering the tradeoff due to the overhead energy consumed due to retransmissions and the latency due to the transmission of
the redundant bits like header and trailer present in a digital data packet. Optimal packet size determination in conventional sensor networks for both uncoded and coded systems were initially proposed in [7]. The authors of [8] proposed a technique to determine the optimal packet size for smart grids using conventional sensor networks. Measurements based on pathloss were obtained to characterize the channel in real time for the smart grid environment. Furthermore, this concept of OPS determination was further extended into CRSNs by the authors in [9] where the sensor nodes with cognitive radio features adapt their packet size dynamically depending on the network condition and other constraints like delay and interference duration to primary users. This was followed with few key literatures in this area like [10] where the authors proposed a dynamic packet optimization and channel selection scheme based on constrained Markov decision process. Majority of the work available in this area emphasizes on the optimization problem formulation to determine the optimal packet size (OPS) and an overview to solve the optimization problem. However the proposed optimization problems are, in general NP hard by nature requiring complex algorithms to solve them. This is not suitable for the sensor nodes which has a very limited computational capacity. To address this challenge we have proposed low complexity suboptimal algorithm in [11] for point-to-point CRSN architecture. We have analyzed the performances of the proposed algorithms based on their required execution time. However, it was observed that with increasing interference from the surroundings or non-cognitive users (-10 dB or more), feasible optimal packet size cannot be obtained due to the network constraints. The main reason for this is the peak power constraint for each individual sensor node of not transmitting beyond 100 mW transmit power. To overcome this challenge, we extend the concept from our previous work and propose a new paradigm based on Multiple Input Multiple Output (MIMO) based CRSNs.

The authors of [12], [13] and [14] have proposed the concept of MIMO based conventional sensor network architecture. The extensive simulation results have revealed significant improvement in the performance of the system in terms of overall energy consumption, transmission range and latency as compared to point-to-point system because MIMO technology enables to exploit diversity and array gain. Therefore, in order to implement a simple MIMO based sensor network architecture when standard space time encoding strategies like Alamouti encoding scheme is applied, the overall transmit power required to attain a specific bit error rate threshold at the receiver end gets divided among the transmit antennas involved in MIMO mode of transmission. This is one of the important motivating factor to incorporate MIMO within the CRSN framework. However, the performance of the conventional MIMO based sensor network architecture when used simultaneously with the cognitive radio framework has not been explored in details. Therefore, exact quantification of the improved performance in terms of overall energy consumption, end-to-end delay and other relevant performance metrics needs to be evaluated and established. Operational criterions like overheads caused due to local intracluster information exchange to form virtual MIMO antenna array, channel sensing, handoff etc. needs to be taken into account. The concept of MIMO technology in general, is applied to enhance the overall throughput of the system by exploiting the diversity and array gain. There are few literatures available like [15] where the authors have coupled this MIMO technology along with an optimal packet size mode of transmission for a single base station with multiple antennas to establish an improved performance in terms of latency, throughput and energy-efficiency. Rate adaption with variable rate m-QAM modulation scheme was adopted as well. However, contribution of this work is mainly aimed towards mobile communication. For MIMO assisted wireless sensor network architecture, the overhead energy consumed due to the cluster formation needs to be taken into account. Therefore, mathematical modelling to determine the OPS will change accordingly. Furthermore, when cognitive radio features are incorporated into the MIMO based sensor network architecture, it would make the mathematical analysis non-trivial. Moreover, the previous work that propose a conventional MIMO based WSNs do not consider the additional interference caused by the surrounding users sharing the same band. This motivated us to introduce the paradigm of cognitive radio along with conventional MIMO-WSNs where the nodes along with MIMO and an optimal packet size mode of transmission would also have the additional feature of cognitive channel sensing and switching based on the network conditions. This helps to counter the effects from the non-cognitive users. Moreover, in our previous works [11], we have established performance improvement for cognitive radio assisted point-to-point WSNs with OPS in terms of overall energy consumption and other key networking metrics. This lead us to the envisage the concept of MIMO assisted cognitive radio enabled wireless sensor networks with optimal packet size (MIMO-CRSN-OPS) which has to best of our knowledge has not been explored so far within the current state of the art. Rigorous analysis through our simulations demonstrate performance improvement of our system as compared to general point to point CRSN architecture.

The main contribution of our paper could be summarized as follows.

1) Firstly, we formulated a joint optimization problem to determine the optimal packet size for a MIMO-Cognitive radio based sensor networks architecture which has not been done so far.

2) Secondly we propose two algorithms to solve the proposed optimization problem. First algorithm is based on Exhaustive Search and the second low complexity suboptimal algorithm is based on solving the conventional Karush-Kuhn-Tucker conditions.

3) Thirdly the MIMO-CRSN framework with optimal packet size mode of transmission is incorporated with a distributed time slotted channel access scheme and a CSMA/CA assisted centralized common control channel based channel access scheme.

This paper is organized as follows. Section II highlights the related work. Section III describes the system model. Section IV describes the different transmission states involved
during cognitive mode of transmission and estimation of the involved channel sensing time for a given detection and false alarm threshold. Section V shows the modelling of the basic optimization problem used to determine the OPS for CRSN. Section VI describes the remodelling and simplification of the optimization problem with variable rate m-QAM based modulation scheme. Section VII describes the proposed algorithm based on exhaustive search and Newton-Raphson assisted KKT-based approach. Numerical results are demonstrated in Section VIII and Section IX concludes the paper.

II. RELATED WORKS

Among the notable works in this area, the authors of [16] presented an exhaustive survey paper on the cognitive radio sensor networks with special emphasis on resource allocation to guarantee quality of service. The concept of Energy Harvesting Cognitive Radio Sensor Networks (EHCRSNs) is proposed by the authors of [17]. In this paper the authors have developed an aggregate network utility optimization framework to design a resource management algorithm based on Lyapunov optimization. In [18] the authors have extended the concept of cognitive radio to propose a cooperative wireless energy harvesting and spectrum sharing for the emerging 5G mobile standards where the secondary users relays and harvest energy from the primary user simultaneously. The authors of [19] have proposed a novel paradigm of heterogeneous cognitive sensor network where two separate categories of sensor nodes are considered within a given network. There are dedicated spectrum sensors which continuously monitor the channel availability and data sensors which scans the area of interest and transmit relevant data from the area of interest. A spectrum scheduling algorithms exclusively for the data nodes jointly enhances the performance of the overall system. In [20] the authors based on queueing theory and classical KKT optimization technique devised an efficient relaying mechanism for a cognitive radio sensor networks architecture where the secondary users maintain a separate queue to relay the data packet of the primary users taking into account the secondary delay, power consumption and admission control acceptance factor. In [21] the authors have proposed a novel cognitive adaptive medium access control scheme (CAMAC) which adapts its channel sensing time and duty cycle to make the system more power efficient. In [22] a spectrum aware cluster based routing protocol is proposed by the authors for multimedia routing. A novel dynamic channel access strategy is proposed in [23] for a clustered CRSN architecture which improves the energy efficiency and throughput both for the intracluster and intercluster communication. It also takes into due consideration appropriate cluster head selection based on available energy and spectrum availability. The authors in [24] proposed a cognitive communication assisted cross layer approach for the smart grid applications. The challenges associated with the harsh smart grid environment and the differential traffic flow are addressed by formulating the problem as a Lyapunov drift optimization. In [25] the classical challenge of spectrum sensing data falsification during joint channel sensing is addressed. An energy efficient collaborative spectrum sensing technique is proposed which considers both independent and collective false spectrum data reporting as a probabilistic measure. Authors of [26], [27], [28] and [29] addresses various challenges associated with the cooperative spectrum sensing and selection of the best nodes within a cluster to improve the detection reliability. In [30] a joint channel access and sampling rate control strategy for energy harvesting CRSNs is proposed. A joint optimization problem based on mix-integer non-linear programming (MINLP) is formulated with fluctuating energy harvesting parameter which maximizes the network utility metric by adapting sampling rate and channel access with energy consumption, capacity and interference as constraints. Among the initial works in the area of packet size optimization for MIMO based systems, the authors of [15] proposed an energy efficient MIMO communication system with packet length adaption with congestion control and delay constraint. M/G/1 queuing theory is used to model the congestion control and delay parameters. A novel event zone to sink aware clustering protocol is proposed by the authors of [31]. The clustering is assumed to be accomplished in two phases which includes determination of the eligible nodes within the event to sink corridor and second phase is the cluster formation. Based on this approach the average re-clustering probability and expected coverage area is determined in this work. In [32], the authors proposed an utility based spectrum access for CRSNs based on random access control. The formulated non-convex optimization problem was solved by the proposed primal decomposition based iterative algorithm. In order to select the best nodes to increasing the channel sensing accuracy in case of joint spectrum sensing , the authors of [4] modelled the mathematical formulation as a binary knapsack problem and solved using dynamic programming taking into account major system constraints like energy consumption and network lifetime. A scalable routing protocol for the CRSNs exclusively for an indoor environment with suitable channel model is proposed in [33]. The simulation results are validated by the results obtained from real time indoor deployment of the sensor nodes. Authors of [34] addressed the issue of optimal spectrum assignment in CRSNs under various network constraints. The modelled MINLP was transformed to a binary linear programming (BLP) using various constraint relaxation techniques. The problem was modelled as mixed integer programming problem. In [35] the authors proposed to improve the spectral efficiency of the CRSNs using in network computation which would minimise the required transmissions and facilitate more simultaneous transmission. A greedy networking algorithm is proposed to improve the quality of service.

III. SYSTEM MODEL

Fig. 1 shows the basic system model of the proposed MIMO-CRSN framework. The network consists of a large number of sensor nodes. Few of the nodes are equipped with cognitive channel sensing and switching feature all of which are operating in the ISM 2.4 GHz band. In addition there are other services like WiFi, Bluetooth, Zigbee and unlicensed
LTE which are also operating in the same unlicensed frequency band causing interference to the cognitive users. These services along with other sensor nodes dedicated for other applications without cognitive feature are collectively termed as non-cognitive users. Whenever an event is triggered, few of the cognitive nodes within the region of event senses the physical phenomenon and transmit the sensed data either through multihop or directly to a distant data gathering server. In order to facilitate \((2 \times 1)\) Alamouti MIMO encoding the cognitive sensor nodes within the event region forms a cluster. In Fig. 1, there are 4 cognitive nodes in a single cluster. This value is denoted as \(M_1\). Among the cognitive nodes in a cluster, there can be a cluster head which is responsible for some additional functionalities like data aggregation or decision fusion in case of cooperative channel sensing. These cognitive nodes initially carries out local information exchange to enable Alamouti encoding. Once the intracluster phase is over, these cognitive nodes do form a virtual antenna array and transmit its data for the long haul communication. In case of a multihop network the data is transmitted either to another intermediate cluster which is not in the event region but do assist to relay the information from the cluster it the event region. In this case there has to be local information exchange at the receiver cluster as well as the Alamouti encoded orthogonal symbols must combined using standard techniques like maximum ratio combining and subsequently decoded to retrieve the originally transmitted symbols. Upon decoding, the retrieved symbols can re-encoded and transmitted to the next available cluster closer to the sink or to the sink directly depending on the network topology. In our analysis for the sake of simplicity we are assuming a single hop system where the information from the cluster in the event region is transmitted to the sink or gateway equipped with a single antenna. It is also assumed that the mean average intracluster distance among the cognitive nodes \((d_{loc})\) is much lesser than the long haul distance.

The cognitive nodes are aware of apriori information like the mean busy time \((l_p)\) and the average idle time \((v_p)\) of the non-cognitive users. The non-cognitive traffic is assumed to be exponentially distributed. Based on these two parameters, the average probability of occupancy \(P_{ro,n}\) and \(P_{ro,f}\) are calculated. These two parameters are estimated to be as \(P_{ro,n} = \frac{l_p}{l_p + v_p}\) and \(P_{ro,f} = 1 - P_{ro,n}\). Depending on these two critical parameters and other network conditions, the cognitive nodes would adapt its various parameters like packet size, modulation level (bits/symbol) and transmit power to improve the performance of the system [9]. It is assumed there are \(C\) data channels accessed simultaneously by both cognitive and non-cognitive users. If there are \(M\) cognitive users in the region of event, effectively \(\left(\frac{M}{M_1}\right)\) number of clusters are in contention to share the \(C\) data channels along with the non-cognitive users are each cluster selects the same channel for data transmission in the proposed MIMO-CRSN system. Moreover, it is assumed that the channel state information (CSI) and noise characteristics are known both to the transmitter and receiver in this paper. The information from the cognitive nodes within the region of event must reach the destination within \(\tau_d\) seconds. Furthermore, the duration of interference caused by these cognitive users to the other surrounding services must be lesser than a certain percentage of the average busy time of the non-cognitive users denoted as \(I_{max}\). In a single cluster only one of the member cognitive nodes senses the channel because it is assumed that the average intracluster distance among the nodes is small as compared to the long haul intercluster or cluster to sink distance.

### IV. Cognitive radio based transmission states

Energy based channel detection scheme is considered for the proposed MIMO-CRSN framework in this paper. The are well established literatures available in this area [36], [37] and [38] which presents a comprehensive mathematical analysis and formulation to estimate various key parameters like the probability of detection \(P_d\), false alarm \(P_f\) and misdetection \(P_{md}\). Probability of detection gives us a measure of how accurately the signals from the primary users or non-cognitive users in our system could be detected correctly by the cognitive users. Misdetection corresponds to the wrong detection of the non-cognitive users whereas false alarm is the measure of wrong detection of the presence of non-cognitive users which leads to a missed opportunity of transmission by the cognitive users. As elaborately explained in our previous work [11] and [9], the respective probabilities of being in these states would be

\[
P_{r_1} = P_{ro,n}P_d
\]

\[
P_{r_2} = P_{ro,n}(1 - P_d)
\]

\[
P_{r_3} = P_{ro,f}P_f,
\]

where \(P_{r_1}, P_{r_2}\) and \(P_{r_3}\) are the respective probabilities of being into the states of detection, misdetection and false alarm. Furthermore, into account the cognitive transmission aspect in case of a distributed channel access scheme, more than a single cognitive user could select the same data channel concurrently.

Fig. 1. Basic system architecture of delay sensitive cognitive radio wireless sensor network
which might lead to co-user interference. The probability of such an event occurring is estimated to be as

$$P_{r_4} = Pr_{off}(1 - P_f) \left[ 1 - \left( \frac{CP_{r_{off}} - 1}{CP_{r_{off}}} \right)^{M-1} \right], \quad (4)$$

where $M$ is the number of contending users in the system and $\left( \frac{CP_{r_{off}} - 1}{CP_{r_{off}}} \right)^{M-1}$ is the probability of more than two users not selecting the same channel. Ergodic behaviour of all the non-cognitive over the $C$ data channels leads to effective $CP_{r_{off}}$ available data channels. In addition there can be a possibility that the cognitive users finds a channel to be vacant and during the transmission duration the non-cognitive user starts transmitting over the same data channel leading to collision and packet drop. The probability of such an event occurring denoted as $P_{r_5}$ would be

$$P_{r_5} = (Pr_{off}(1 - P_f) - Pr_{4})Pr(V_p \leq \frac{l_s}{R}) \quad (5)$$

$$P_{r_5} = (Pr_{off}(1 - P_f) - Pr_{4}) \int_{\frac{l_s}{R}}^{\infty} \frac{1}{v_p} e^{-\frac{v_p}{v_{tar}}} dt$$

$$P_{r_5} = (Pr_{off}(1 - P_f) - Pr_{4}) \left( 1 - e^{-\frac{l_s}{v_{tar}} v_p} \right)$$

where, $v_p$ is the average idle time, $R$ is the data rate, $l_s$ is the packet size in bits, $P_{r_{off}}$ is the probability of unoccupancy by the non-cognitive users, $P_f$ is the false alarm. This state is feasible if and only if the state of co-users interference ($Pr_4$) as estimated in (4) doesn’t occur. Finally the state of successful transmission is achieved when both state of co-user interference and collision doesn’t occur and the idle time of the non-cognitive users is greater than the packet transmission duration which results into

$$P_{r_6} = e^{-\frac{l_s}{v_{tar}} v_p} (Pr_{off}(1 - P_f) - Pr_{4}). \quad (6)$$

In addition, during the state of Misdetection the interference to the non-cognitive users could last for entire duration of the cognitive transmission or the cognitive users could decide to vacate the channel during the transmit duration of the cognitive users. The probability of such an event occurring would be $Pr(L_p \leq \frac{l_s}{R}) = (1 - e^{-\frac{l_s}{v_{tar}} v_p})$. Similarly the probability of such event not occurring during the state of misdetection or in other words the non-cognitive user’s transmission lasting throughout the duration of the cognitive packet transmission would be $1 - Pr_{off}(1 - e^{-\frac{l_s}{v_{tar}} v_p})$. Similarly the probability of such event not occurring during the state of misdetection or in other words the non-cognitive user’s transmission lasting throughout the duration of the cognitive packet transmission would be $1 - Pr_{off}(1 - e^{-\frac{l_s}{v_{tar}} v_p})$. Since $Pr_{on} + Pr_{off} = 1$ thus, the probability that there will be collision and packet drop for cognitive users during the state of collision would be $Pr_{on} + Pr_{off} e^{-\frac{l_s}{v_{tar}} v_p}$.

As shown by the authors of [37], depending on the gaussian probability distribution function of the received interference power from the non-cognitive users and the central limit theorem, the probability of detection ($P_d$) and false alarm ($P_f$) is estimated to be as

$$P_d = Q\left( \frac{\lambda}{\sigma_u^2} - \gamma_{pr} - 1 \right) \frac{\tau_s f_s}{2\gamma_{pr} + 1} \quad (7)$$

$$P_f = Q\left( \frac{\lambda}{\sigma_u^2} - 1 \right) \sqrt{\tau_s f_s} \quad , \quad (8)$$

where $\tau_s$ is the channel sensing duration, $\gamma_{pr}$ is the received signal strength at the cognitive transmitter from the non-cognitive users, $\lambda$ is the detection threshold of the energy based detector and $\sigma_u^2$ is the total noise power. Taking $\lambda$ as an explicit variable the sensing time required for a specific $P_d$ and $P_f$ threshold for a specific $\gamma_{pr}$ turns out to be

$$\tau_s = \frac{1}{2B^2 \gamma_{pr}} \left[ Q^{-1}(P_f) - Q^{-1}(P_d) \sqrt{2\gamma_{pr} + 1} \right]^2 \quad (9)$$

In the remaining part of this paper $\gamma_{pr}$ is denoted as $SNR_{pr}$ and $\sigma_u^2 = N_0B$ where $N_0$ is the noise power spectral density whose value is -171 dBm/Hz and $B$ is the channel bandwidth at 1 MHz.

V. PROBLEM FORMULATION FOR MIMO-CRSN ARCHITECTURE

A. Formulation and analysis of the cost function

Determination of optimal packet size in MIMO-CRSN architecture involves formulation of a cost function which takes into account the overall energy efficiency ($\epsilon_{mimo}$) and the packet reliability ($r_{mimo}$) of the system. It is dependent on the packet size $l_{mimo}$ symbols. Furthermore, for MIMO based architecture the additional overhead energy consumed during the intracluster phase at the transmitter end to form the virtual MIMO antenna array denoted as $E_{local}$ in order to facilitate Alamouti encoding plays a significant role that needs to be considered. Moreover, factors contributed by the cognitive functionalities such as channel sensing, channel handoff, channel decision alongside energy consumed during
transient phases like activation from sleep to wake up mode must also be included. Based on these collective factors an appropriately designed cost function could provide an optimal packet size for MIMO-CRSN architecture.

For MIMO-CRSN architecture the data packets from a given number multiple nodes in a single cluster participating in data communication \( M_t \) must transmit its data simultaneously and in a synchronized manner to the sink node for proper decoding of the orthogonally encoded Alamouti bit streams from \( M_t \) sensor nodes. Therefore, unlike point-to-point CRSN architecture where a single data packet from its corresponding sensor node is used to estimate the OPS, in MIMO CRSN node data packets from \( M_t \) must be considered jointly and to be treated as a single data superpacket denoted as \( l^\text{MIMO} \). In Fig. 3, an Alamouti encoded \((2 \times 1)\) MIMO-CRSN packet structure is described. For example, for two sensor nodes \((M_t = 2)\) sensor nodes in a given cluster, it is assumed that each node has four independent data symbols \( x_i \) and \( \bar{x}_i \) for node 1 and 2 respectively where \( i = \{1, 2, 3, 4\} \) and three independent header symbols \( x_{hj} \) and \( \bar{x}_{hj} \) where \( j = \{1, 2, 3\} \) that is to be transmitted to the sink node. Header segment of a data packet contains critical information like the node id that is to be transmitted to the sink node. Header segment of a data packet contains critical information like the node id that is to be transmitted to the sink node. For Alamouti encoded symbols, 6 of which attributed by the encoded symbols. Therefore, as shown in Fig. 3, the MIMO data packet from each of the sensor node is likely to have 14 Alamouti encoded data symbols, 6 of which attributed by the encoded header symbols and remaining 8 encoded data symbols. Both node 1 and 2 will have to transmit these 14 orthogonally encoded symbols simultaneously in a synchronized manner for proper detection at the sink. The sink could be the final gateway itself in case of a single hop scenario or it could be the cluster head of an intermediate cluster. Therefore, in our example the packet size for each node operating under MIMO mode \( l^\text{MIMO} = 14 \) symbols. Our objective is therefore to obtain the optimal size of \( l^\text{MIMO} \) denoted from now on as \( l^\text{opt} \) which is the optimal packet size for our proposed MIMO-CRSN architecture.

For the proposed MIMO-CRSN architecture, to simplify our further analysis we assume the long haul transmission energy consumption per independent symbol to attain a specific bit error rate threshold denoted as \( k^\text{MIMO} \) as shown by the authors in [12]. It is however important to mention that although \((14 + 14) = 28\) Alamouti encoded symbols are transmitted 14 symbols each from node 1 and 2 respectively, the overall energy consumption for the transmission of the MIMO superpacket generated by \( M_t = 2 \) participating nodes in the cluster will be \( k^\text{MIMO} \times 14 \) and not 28. This is due to the fact that when we consider energy consumption for the independent symbols, while using full rate code like \((2 \times 1)\) Alamouti code the effect of transmitting the conjugate symbols \( \bar{x}^* \) and \( \bar{x}^* \) in the even time slots is already included in the \( k^\text{MIMO} \). Since in the MIMO superpacket there are 14 indepen-
As per authors of [40], energy consumed for channel switching (\(E_{h.f}\)) in practical applications for a relaxed scenario when the channel center frequencies are close by is around 2 mJ. Therefore average energy consumed for channel handoff for CR architecture turns out to be \(Pr_{su}E_{h.f}\).

In MIMO-CRSN architecture the data packets from \(M_t\) nodes within the cluster must be transmitted and received in a simultaneous and synchronized manner. It is assumed that drop of data packet from any single \(M_t\) user within the cluster would lead to failed detection of the entire MIMO superframe. Therefore, for correct detection the packets from each of the \(M_t\) users must reach and detected correctly at the receiver. For each node, the packet error rate (\(PER_{\text{single}}\)) and reliability (\(r_{\text{single}}\)) is calculated to be as

\[
PER_{\text{single}} = 1 - (1 - Pr_c)^{l_s}, \quad (18)
\]
\[
r_{\text{single}} = 1 - PER_{\text{single}}, \quad (19)
\]

where \(Pr_c\) is the average bit error rate of the MIMO-CRSN model and \(l_s\) is the packet size.

Therefore, the overall reliability of the MIMO superpacket is estimated to be as

\[
r_{\text{mimo}}(l_s) = r_{\text{single}}^{M_t} \quad (20)
\]
\[
r_{\text{mimo}}(l_s) = (1 - PER_{\text{single}})^{M_t} = (1 - Pr_c)^{M_t l_s}. \quad (21)
\]

Therefore, the cost function is simplified to

\[
\eta_{\text{mimo}}(l_s) = \frac{k_1(l_s - M_t h)}{k_1 l_s + k_{loc}(l_s - M_t h) + E_{tot}(1 - Pr_c)^{M_t l_s}}. \quad (22)
\]

The motivation to consider the cost function as a multiplicative factor of the energy-efficiency \(\eta_{\text{mimo}}\) and reliability \(r_{\text{mimo}}\) is owes to fact that the formulated cost function in this case will be concave with respect to the packet length \(l_s\) with an unique global maxima. Other modes of designing the same like weighted-sum approach would not guarantee a concave behaviour of the cost function which is needed to find the optimal packet size both mathematically and intuitively. Analyzing the cost function further, the first and the second derivative of the cost function yields

\[
\eta_{\text{mimo}}(l_s)' = Z_1(l_s) Z_2(l_s), \quad (23)
\]

where \(Z_1(l_s)\) and \(Z_2(l_s)\) are dummy variables evaluated to be as

\[
Z_1(l_s) = \frac{E_{tot}k_1 + k_1^2 M_t h}{(k_1 + k_{loc})l_s + E_{tot} - k_{loc} M_t h} + \frac{k_1(l_s - M_t h) M_t ln (1 - Pr_c)}{k_1 l_s + k_{loc}(l_s - M_t h) M_t ln (1 - Pr_c)} + (24)
\]
\[
Z_2(l_s) = \frac{1}{(k_1 + k_{loc})l_s + E_{tot} - k_{loc} M_t h}. \quad (25)
\]

By replacing \(E_{tot} - k_{loc} M_t h = Z_3\), the double derivative of the cost function turns out to be

\[
\eta_{\text{mimo}}(l_s)'' = Z_1(l_s)' Z_2(l_s) + Z_2(l_s) Z_1(l_s). \quad (26)
\]

The double derivative of the dummy variables \(Z_1(l_s)\) and \(Z_2(l_s)\) are calculated to be as

\[
Z_1(l_s)' = ln (1 - Pr_c) M_t k_1 - \frac{(E_{tot} k_1 + k_1^2 M_t h)(k_1 + k_{loc})}{(k_1 + k_{loc})l_s + Z_3} \quad (27)
\]
\[
Z_2(l_s)' = \frac{Z_4(l_s)}{(k_1 + k_{loc})l_s + Z_3}^2, \quad (28)
\]

where

\[
Z_4(l_s) = ((k_1 + k_{loc})l_s + Z_3) ln (1 - Pr_c) M_t (1 - Pr_c)^{M_t l_s} (k_1 + k_{loc})(1 - Pr_c)^{M_t l_s}. \quad (29)
\]

Considering the fact that \(Pr_c < 1\), it implies \(ln (1 - Pr_c) \approx 0\). Using (27), (28) and (29) it can be proven that \(\eta_{\text{mimo}}(l_s) < 0\). Therefore, the cost function \(\eta(l_s)\) is a concave function with respect to the packet size \(l_s\) with an unique maxima. Therefore, the optimal packet size turns out to be \(l_s^*\) which maximizes the following cost function as long as all the posed constraints criteria are satisfied.

\[
\max_{l_s} \eta_{\text{mimo}}(l_s) = \frac{k_1(l_s - M_t h)}{(k_1 + k_{loc})l_s + E_{tot} - k_{loc} M_t h} (1 - Pr_c)^{M_t l_s}. \quad (30)
\]

For a fixed average BER \(Pr_c\), packet size \(l_s\) and \(E_{tot}\), differentiating the cost function \(\eta_{\text{mimo}}\) with respect to \(k_1\) leads to

\[
\frac{\partial \eta_{\text{mimo}}}{\partial k_1} = \frac{k_1 l_s - M_t h + Z_3 l_s (l_s - M_t h)}{(k_1 + k_{loc})l_s + Z_3} (1 - Pr_c)^{M_t l_s}. \quad (31)
\]

As \(l_s > M_t h\), from the basic assumption, it can be proven that the cost function \(\eta_{\text{mimo}}\) is an increasing function of \(k_1\) for fixed value of \(l_s\), \(Pr_c\) and other parameters remaining constant.

Energy consumption per bit \(k_1\) depends upon the power consumed by the power amplifiers and the circuit components of the transceiver nodes [13]. In case if m-QAM modulation scheme is used, this energy consumption per bit will depend on the modulation level \(b\) as the data rate \(R\) will depend on \(b\) for a fixed symbol rate for the system \(R_s\).

\[
k_1 = (P_{PA} + P_c) \frac{1}{b R_s}. \quad (32)
\]

Energy consumption per bit \(k_1\) is a function of the modulation level \(b\). It has been extensively discussed in our previous work and previous established literatures that in case of variable rate modulation scheme, for relatively smaller distances and for given fixed BER threshold, the energy consumption per bit \(k_1\) will exhibit a convex behaviour with respect to the modulation level \(b\). This behaviour holds true both for point to point and MIMO systems alike. The rationale behind such behaviour is contributed by the fact that increase in the modulation level increases the data rate since \(R = b R_s\) and minimizes the transmit duration. Upto a specific value of \(b\) say...
the transmit duration will be dominant factor therefore, the value of $k_1$ would decrease. However, the power consumption of the power amplifier $P_{PA}$ is an increasing function with respect to $b$. Beyond the value of $b^*$, $P_{PA}$ will become the dominant factor instead of the transmit duration thus resulting into an increased value of $k_1$.

In the subsequent section it will be shown that the average BER $T_b$ of the proposed MIMO-CRSN architecture will depend on the probabilities of being in the state of misdetection and collision. Under these cases the data packet from the cognitive experience interference either from the non-cognitive users or from other cognitive users selecting the same channel. The probabilities of these states directly depends upon the transmit duration of the data packet or in other words the packet length ($l_1$) as shown in Section IV. Therefore, the energy consumption per bit $k_1$ to attain a specific BER threshold will be dependant on the packet size along with the modulation level thus making it $k_1(l_1, b)$, a function of both $l_1$ and $b$. It is already shown from (26) to (29), the cost function $\eta_{\text{mimo}}(l_1)$ will be concave function of the packet size $l_1$ for a fixed $T_b$, fixed modulation level $b$ and its corresponding energy consumption per bit $k_1(b)$. However, in our case since $k_1$ is also dependant on $l_1$ makes further analysis non trivial. Furthermore, it is observed that for fixed modulation level $b$, the concavity of the cost function $\eta_{\text{mimo}}(l_1, b)$ will be preserved even though $k_1(l_1, b)$ now being dependant on the packet size. It is also observed that $k_1(l_1)$ will be an increasing function of $l_1$ for fixed value of $b$. However, the magnitude of increase in the $k_1$ value with increasing $l_1$ for a fixed $b$ is much smaller to affect the very nature of the cost function $\eta$. It is proven $\eta_{\text{mimo}}(l_1)^{\prime\prime} < 0$ in our appendix section. Based on this argument for a given modulation level $b$, the optimal packet size $l_1^*$ which maximizes the cost function could be written as

$$T_b^* = \left\{ \max_{l_1} \eta_{\text{mimo}}(l_1, b) \right\} \forall b,$$  \hspace{1cm} (33)

where $T_b^*$ is the set of optimal packet size for different modulation levels. Along finding the optimal packet size for the MIMO-CRSN architecture, it is also our objective to ensure the system must consume minimum energy which is one of the essential requisite for any sensor network architecture. This essentially leads us to the conclusion that the selection of the appropriate modulation level $b$ holds the key for which the the energy consumption per bit $k_1(b)$ would be minimum. From (31) it is proven that the cost function $\eta_{\text{mimo}}$ will be an increasing function of $k_1$ when packet length and BER threshold is fixed. Moreover, the authors of [13] has shown that $k_1$ will show a convex behaviour with respect to $b$ for a fixed $T_b$. Therefore, the optimal modulation level $b^*$ must be selected among the set of given modulation levels for which the energy consumption per bit $k_1$ would be minimum. Based on this and the previous property it can be easily concluded that the cost function $\eta_{\text{mimo}}$ too will exhibit a convex behaviour with respect to modulation level $b$. For individual modulation levels, its corresponding optimal packet size is already estimated in (33) as $T_b^*$

$$b^* = \left\{ \min_b \eta_{\text{mimo}}(l_1^*, b) \right\} \forall b,$$  \hspace{1cm} (34)

Therefore, combining (33) and (34), the cost function to obtain the optimal packet size and optimal modulation level could be written as a min-max problem

$$\min_b \left\{ \max \eta_{\text{mimo}}(l_1, b) = \frac{k_1(l_1, b)(l_1-M_1b)(1-T_b^*)^{M_1s}}{(k_1(l_1, b)+\xi_{\text{mimo}}l_1, b+\xi_3)\nu b} \right\},$$  \hspace{1cm} (35)

where $Z_3 = E_{\text{tot}} - k_{\text{tot}}M_1b$.

B. Modelling of the total interference time constraint for the non-cognitive user

The duration for which the non-cognitive users experiences interference from the cognitive secondary users plays an important role for any cognitive radio based network. As explained in [11], [9], it depends upon the probabilities of being into the state of misdetection and collision. However, in case of MIMO-CRSN architecture the number of nodes in a given cluster ($M_1$) will transmit its data sharing the same channel. Therefore, if there are $M$ total number of users participates in data transmission over $C$ data channels, there will be effectively $\frac{M_c}{M}$ number of users among which there will be contention within $C$ data channels. Therefore, the probability of co-user selection $Pr_{c}$ for MIMO-CRSN architecture will now turn out to be

$$Pr_{c} = Pr_{\text{off}}(1-Pr) \left\{ 1 - \left(Pr_{\text{off}} - 1 \right) \left(\frac{M}{1-M_c} \right) \right\}. \hspace{1cm} (36)$$

Based on $Pr_{c}$, the probability of collision will be

$$Pr_{c} = \{ Pr_{\text{on}}(1-Pr_{c}) - Pr_{c} \} \frac{\gamma_{\text{c}}}{Pr_{c}} \hspace{1cm} (37)$$

As explained in [9] and [11], during the state of misdetection the duration of interference with the non-cognitive users could last either for the entire duration of the packet transmission of the cognitive user or the non-cognitive user may decide to vacate the channel during the transmission phase of the cognitive user. In the later case the non-cognitive users will not experience any interference from the cognitive users. Based on above, the total average duration of interference cause to the non-cognitive users for the proposed MIMO-CRSN architecture boils down to

$$Pr_{c} = \frac{l_1 Pr_{\text{on}}(Pr_{\text{off}} + Pr_{\text{off}} e^{-\gamma_{\text{c}}})) + Pr_{\text{on}}Pr_{c} Pr_{\text{in}}}{Pr_{\text{c}} + Pr_{c} Pr_{\text{in}}}.$$  \hspace{1cm} (38)
where $R = bR_s$ is the data rate of the system.

Therefore, the ratio of the average interference time to the average busy transmission time of the non-cognitive user denoted by $I_{nc}$ will be,

$$I_{nc}^{\text{mimo}}(l_s, b, P_d, P_f, M_t) = \frac{I_{nc}^{\text{mimo}}}{I_p}. \quad (39)$$

$I_{nc}^{\text{mimo}}$ must be lesser than $I_{\text{max}}$ which is application specific constraint threshold. Therefore, from (38) and (39) the constraint could be written as

$$c_1^{\text{mimo}}(l_s, b, P_d, P_f, M_t) = I_{nc}^{\text{mimo}}(l_s, b, P_d, P_f, M_t) - I_{\text{max}} \leq 0 \quad (40)$$

C. Modelling of the end to end delay constraint for the cognitive user

An infinite Stop and Wait ARQ scheme for the proposed MIMO-CRSN architecture is considered which is used to calculate the average time to transmit each data packet and takes into account the retransmission overheads [9], [11]. For $M$ cognitive users participating in data communication each of which has $K$ bits to transmit, there are $M_t$ sensor nodes in each cluster. Therefore, there are $\frac{M}{M_t}$ number of clusters in the MIMO-CRSN network. For each of these clusters, there has to be local intracluster information exchange to enable Alamouti MIMO encoding. This consumes overhead delay. Furthermore, due to the cognitive feature of the network just like point-to-point CRSN architecture there will be additional delay incurred due to channel sensing, channel decision, handoffs, processing delay at each hop for each packet and transient delay due to wake up and sleep functionalities of the sensor nodes.

The average packet error rate for the for the MIMO-CRSN model could be directly obtained from (21) as

$$PER^{\text{mimo}}(l_s) = 1 - r^{\text{mimo}}(l_s). \quad (41)$$

Based on the Stop and Wait ARQ retransmission mechanism the average delay for the long haul transmission will be

$$E(T) = T_s \left(1 + \frac{PER^{\text{mimo}}}{1 - PER^{\text{mimo}}} \right) \quad (42)$$

$$E(T) = T_s \left(1 + \frac{1 - (1 - P_e)^{M_t l_s}}{(1 - P_e)^{M_t l_s}} \right),$$

where $T_s = \left(\frac{l_s}{R} + \tau_{\text{add}}\right)$ where $\frac{l_s}{R}$ is the data transmission time.

$$\tau_{\text{add}} = \tau_d + \tau_s + \tau_{\text{dec}} + \tau_{hf} + \tau_{sl/wk} \quad (43)$$

is the additional delay caused due to processing at each hop ($\tau_d$), channel sensing time ($\tau_s$), handoff ($\tau_{hf}$), spectrum decision ($\tau_{\text{dec}}$), and transient time for the receiver to wake up from sleep to active model. Values of $\tau_{hf}$ and $\tau_{\text{dec}}$.

$$\tau_{\text{total}} = \frac{MK\pi}{(l_s - M_l h_s)} \left\{ \left(1 + M_t P_e l_s \right) \left(\tau_{\text{total}} + \frac{l_s}{R}\right) + \frac{l_s - M_l h_s}{R_{\text{loc}}} \right\},$$

where $R_{\text{local}} = b_{\text{loc}} R_s$ is the data rate of the system during intracluster communication, $b_{\text{loc}}$ is the corresponding modulation level, $\pi$ is the average number of hops and $k = \frac{\tau_d}{MK\pi}$ where $\tau_d$ is the total end to end delay threshold. For much shorter distances for intracluster communication, the modulation level is usually high [12].

The total delay $\tau_{\text{total}}$ must be less than or equal to a specific end to end delay threshold $\tau_d$. Therefore, (44) could be rewritten as

$$c_2^{\text{mimo}}(l_s) = \tau_{\text{total}} - \tau_d \leq 0 \quad (45)$$

$$c_2^{\text{mimo}}(l_s) = \frac{M_k P_e^2}{b} l_s^2 + \left(1 - \frac{1}{b_{\text{loc}}} + M_t P_e \tau_{\text{total}} R_s - k R_s \right) + \left(M_t k R_s + \tau_{\text{total}} R_s - \frac{M_l h_s}{b_{\text{loc}}} \right) \leq 0 \quad (46)$$

D. Constraint on the transmit power

Similar to that of point-to-point CRSN network in [11], it is assumed that each sensor node has maximum peak power constraint of 20 dBm which is equal to 100 mW. Similar constraint holds true even for proposed MIMO-CRSN architecture but because of the Alamouti encoding, the transmit power is equally spitted among the $M_t$ transmit antennas corresponding to each node in the cluster. Although this scales down the received signal to noise ratio at the receiver but due to the inherent transmit diversity of the Alamouti encoding, there will be significant improvement in the performance which is shown through our extensive simulation results. Furthermore, in order to attain a specific average BER ($P_e$), the required transmit power would depend on the modulation level ($b$) and packet size ($l_s$). Therefore, the transmit power constraint could be written as

$$c_3^{\text{mimo}}(l_s, b, P_d, P_f, P_e) = \frac{P_{\text{out}}(l_s, b, P_d, P_f, P_e)}{M_t} - 0.1 \leq 0. \quad (47)$$

E. Constraint on the average BER

As modelled in point-to-point CRSN architecture, a constraint on the average BER $P_e$ for our MIMO-CRSN architecture is considered. The rationale behind this is the fact that cognitive radio based network has various transmission sates
each with its own instantaneous BER based on the received SNR and SINR (discussed in the next section). To ensure a certain level of transmission reliability of the system and for the sake of mathematical simplification a constraint on the average BER is a reasonable technique that has already been established. For multihop scenario, the end to end average BER will turn out to be

\[ 1 - (1 - \mathcal{T}_e)^n \leq \mathcal{T}_{\text{eth}}, \quad (48) \]

where \( \mathcal{T}_e \) is the average BER of each hop and \( n \) is the average number of hops. This must be less than or equal to a specific threshold \( \mathcal{T}_{\text{eth}} \). In case of single hop the average BER will be simply \( \mathcal{T}_e \leq \mathcal{T}_{\text{eth}} \).

F. The optimization problem

To guarantee protection of the non-cognitive users and to maximize the transmission opportunity by the cognitive users, the probability of detection \( P_d \geq P_a \) and \( P_f \leq P_l \) where \( P_d = 0.9 \) and \( P_f = 0.1 \) which is the benchmark as per any cognitive radio specifications. Based on the on this and above subsections the optimization problem to determine the optimal packet size boils down to

\[
\begin{align*}
\min_{b} & \quad \eta_{\text{mimo}}(l_s, b, P_d, P_f, \mathcal{T}_e, M_t) \\
\text{subject to} & \quad \epsilon_1^{\text{mimo}}(l_s, b, P_d, P_f, M_t) \leq 0 \\
& \quad \epsilon_2^{\text{mimo}}(l_s, b, P_d, P_f, \mathcal{T}_e, M_t) \leq 0 \\
& \quad \epsilon_3^{\text{mimo}}(l_s, b, P_d, P_f, \mathcal{T}_e, M_t) \leq 0 \\
& \quad \mathcal{T}_e \leq 1 - e^{\frac{1}{n} \log(1 - \mathcal{T}_{\text{eth}})} \\
& \quad P_d \geq \hat{P}_d \\
& \quad P_f \leq \hat{P}_f \\
& \quad 100 < l_s < 1000, b \in \{2, 3, 4, \ldots, 10\},
\end{align*}
\]

(49a) where both \( b \) and \( l_s \) are discrete integers.

VI. Determination of the average BER under VARIABLARTE M-QAM AND REMODELLING OF THE OPTIMIZATION PROBLEM

The transmit power \( (P_{\text{out}}) \) is determined based on the average BER \( \mathcal{T}_e \). Therefore the energy consumption Average probability of error \( \mathcal{T}_e \) will depend on the received SNR without interference from the non-cognitive user \( (\gamma_a) \) and received signal to interference and noise ratio which is the SINR \( (\gamma_b) \) both in terms of normalized bit energy to noise ratio \( \frac{E_b}{N_0} \).

\[
\gamma_a = \frac{||H||^2}{P_{\text{rec}} N_0 R} \quad (50)
\]

\[
\gamma_b = \frac{||H||^2}{P_{\text{rec}}} \left( \frac{N_0 + P_{\text{nc}}}{N_0} \right)R \quad (51)
\]

\[
\gamma_b = \frac{\gamma_a}{(N_0 + P_{\text{nc}})} \quad (52)
\]

where \( P_{\text{rec}} \) is the received power from the cognitive transmitter to the cognitive receiver, \( P_{\text{nc}} \) is the power received at the cognitive receiver from the non-cognitive user as interference and \( g \) being instantaneous channel gain component with Rayleigh distribution. Again, since \( R = b R_s \) therefore, both \( \gamma_a \) and \( \gamma_b \) will depend on the modulation level \( b \). In Section III A, it is already shown that \( \gamma_a = \frac{\sigma^2_a}{\sigma^2} \) where \( \sigma^2_b \) is the signal power received at the receiver end, \( \sigma^2_a = N_0 B \) is the total noise power where \( \frac{N_0}{N_0} \) is single sided power spectral density and \( B \) is the bandwidth of the channel. Thus \( \sigma^2_a = \gamma_b (N_0 B) \). \( \sigma^2_b \) is now denoted as \( P_{\text{nc}} \) in this paper.

Power received at the cognitive receiver will depend upon the transmit power and the corresponding system and network configuration which includes the pathloss, link margin, antenna gains and system implementation losses etc. \( P_{\text{rec}} \) will be dependant on the transmit power and based on Friss law of pathloss, \( P_{\text{rec}} \) can be easily calculated to be as

\[
P_{\text{rec}} = P_{\text{out}} + G_{t4B} + G_{r4B} + K_{pl4B}
\]

\[-10 \log_{10} \left( \frac{d_{ss}}{d_0} \right) - N_{f4B} - M_{4B}, \]

where \( K_{pl} = 20 \log_{10} \left( \frac{d_{out}}{d_0} \right) \) is the pathloss component, \( G_{t/r} \) are the gains of the transmit and receive antennas, \( N_f \) is the noise figure, \( M_l \) is the link margin, \( \delta \) is the pathloss exponent and \( d_0 = 1 \text{ m} \) is the reference distance.

Taking the absolute value of the \( P_{\text{rec}} \) and substituting in (50) and (52) we can obtain the instantaneous SNR and SINR. Taking expectation operator \( \mathbb{E}(\cdot) \) of \( \gamma_a \) and \( \gamma_b \) yields the average received SNR and SINR \( \gamma_a \) and \( \gamma_b \) which is used to estimate the average BER of the system.

As explained in the earlier section that the average BER is dependant on the probability of different cognitive transmission states, the instantaneous BER needs to be calculated for different states where cognitive nodes transmits and the instantaneous BER has to be averaged over the pdfs of the received SNR and SINR to obtain the average BER. Let \( \zeta(\gamma_a) \) and \( \zeta(\gamma_b) \) be the corresponding BERs for the SNR \( (\gamma_a) \) and SINR \( (\gamma_b) \) respectively. Therefore, the instantaneous BER for different cognitive states can be estimated as

\[
\zeta(\gamma_a) = \frac{P_{\text{on}}}{P_{\text{off}}} \left( 1 - e^{-\frac{\gamma_a}{\sigma^2}} \right),
\]

\[
\zeta(\gamma_b) = \frac{P_{\text{off}}}{P_{\text{on}}} \left( 1 - e^{-\frac{\gamma_b}{\sigma^2}} \right),
\]

\[
\text{Mis detection (\zeta(\gamma)) : } \left( P_{\text{on}} + P_{\text{off}} e^{-\frac{\gamma_a}{\sigma^2}} \right) \zeta(\gamma_a) + \frac{P_{\text{off}}}{P_{\text{on}}} \left( 1 - e^{-\frac{\gamma_b}{\sigma^2}} \right) \zeta(\gamma_b) \quad (54)
\]

\[
\text{Co - selection (\zeta(\gamma)) : } \zeta(\gamma_a) + \zeta(\gamma_b) \quad (55)
\]

\[
\text{Collision (\zeta(\gamma)) : } P_{\text{on}} \zeta(\gamma_a) + P_{\text{off}} \zeta(\gamma_b) \quad (56)
\]

\[
\text{Successful transmission (\zeta(\gamma)) : } \zeta(\gamma_a), \quad (57)
\]

where \( \zeta(\gamma), \gamma \in \{\gamma_a, \gamma_b\} \) is the BER expression for the variable m-QAM modulation scheme given by

\[
\zeta(\gamma) = \frac{4}{b} \left( 1 - \frac{1}{2^b} \right) Q \left( \frac{\sqrt{3b}}{(2^{b-1}) M_t} \right),
\]

(58)

Instantaneous BERs obtained in (54) to (57) needs to be weighted with the probabilities of its corresponding transmission states as shown in (13), (16) to (18). Therefore, the total instantaneous BER is calculated to be

\[
\zeta_{\text{total}} = \frac{P_{\text{on}} \zeta(\gamma_a) + P_{\text{off}} \zeta(\gamma_b)}{P_{\text{on}}(1 - P_d) + P_{\text{off}}(1 - P_f)},
\]

(59)
Substituting (54 to (57) in the above equation (59) and by simplifying we obtain
\[
\zeta_{total} = \zeta(\gamma_a) + \Omega \{ \zeta(\gamma_b) - \zeta(\gamma_a) \},
\]
where
\[
\Omega = \frac{Pr_2(Pr_{on} + Pr_{off} \cdot \frac{M_t}{M_s}) + P_{M}^{mimo} + Pr_{on}Pr_{off} \cdot \frac{M_s}{M_t}}{Pr_{off}(1 - P_f) + Pr_{on}(1 - P_d)}.
\]

The total instantaneous BER will turn out to be
\[
\zeta_{total} = \frac{4}{b} \left( 1 - \frac{1}{2^b} \right) \left[ (1 - \Omega) \left\{ \frac{3b}{\sqrt{(2^b - 1)M_t}} \right\}^\gamma \right. \\
\quad + \left. \Omega \left\{ \frac{3b}{\sqrt{(2^b - 1)M_t}} \right\}^{\gamma_b} \right],
\]
(60)

The average BER of the MIMO-CRSN system will be
\[
\overline{P_c} = E_{\gamma}(\zeta_{total}),
\]
where \(E(\cdot)\) is the expectation operator.

Therefore, the average BER \(\overline{P_c}\) turns out to be
\[
\overline{P_c} = \frac{4}{b} \left( 1 - \frac{1}{2^b} \right) \left[ (1 - \Omega) E_{\gamma_a} \left\{ \frac{3b}{\sqrt{(2^b - 1)M_t}} \right\}^\gamma \right. \\
\quad + \left. \Omega E_{\gamma_b} \left\{ \frac{3b}{\sqrt{(2^b - 1)M_t}} \right\}^{\gamma_b} \right],
\]
(61)

where from [41], [42].
\[
E_{\gamma} \left\{ \frac{3b}{\sqrt{(2^b - 1)M_t}} \right\} = \left[ \frac{1}{2} \right] \left[ 1 - \sqrt{\frac{3b\gamma}{2 + \frac{3b\gamma}{3b\gamma}}} \right] \frac{L}{2^b} \sum_{i=0}^{L-1} \left[ \frac{1}{2^b} \right]^{L-1+i} \left[ 1 - \sqrt{\frac{3b\gamma}{2 + \frac{3b\gamma}{3b\gamma}}} \right],
\]
(62)

In the above equation (65), \(L = (M_t \times N_r)\) which is the diversity order. For \(\gamma \in \{\gamma_a, \gamma_b\}\) taking into account the Frobenius norm of the MIMO channel matrix [42]. In our simulation

Since \(\gamma_a >> 1\) and \(\gamma_b > 1\) at very high values of \(\gamma_a\)
\[
E_{\gamma} \left\{ \frac{3b}{\sqrt{(2^b - 1)M_t}} \right\} = \left( 2L - 1 \right) \frac{1}{2^b} \left\{ \frac{3b}{(2^b - 1)M_t} \right\}^{\gamma_b},
\]
(63)

From (36), (37) and (61), since \(\Omega\) is a function of packet size \(l_s\) therefore, the the average BER \(\overline{P_c}\) will be a function of packet size. Furthermore, it is also observed that \(\Omega\) depends on the probability of detection \(P_d\) and probability of false alarm \(P_f\) (61). It has been shown by the authors of [37] that for a given probability of detection threshold \(P_d\), the probability of false alarm \(P_f\) will be a monotonically decreasing function with respect to the channel sensing time \(\tau_s\). Since our network is a delay sensitive network and longer channel sensing duration consumes additional overhead energy (14), the probability of detection and false alarm is fixed to a specific threshold \(\tilde{P_d}\) and \(\tilde{P_f}\) and treat them as equalities instead of the inequalities as shown in the constraints of the formulated optimization problem (49f) and (49g). This essentially leads to further simplification of the the optimization problem.

Using the equation above, now we can easily calculate the received SNR at the cognitive receiver and the corresponding transmit power required to attain specific BER threshold of \(\overline{P_{eth}}\) from (53) and (66),
\[
P_{out}(l_s, b, \tilde{P_d}, \tilde{P_f}, \overline{P_{eth}}, M_t, N_r) \leq \frac{(4\pi)^2 d_{th}^2 M_t N_r}{G_t G_r \lambda^2} \gamma_a N_0 R.
\]
(64)

Similarly, since energy consumed per bit \(k_1\) is depending directly on the transmit power \(P_{out}\). From (35) and (67),
\[
k_1(l_s, b, \tilde{P_d}, \tilde{P_f}, \overline{P_{eth}}) \leq \{(1 + \alpha)P_{out} + P_d\} \frac{1}{R}.
\]
(65)

The initial cost function (36) now becomes,
\[
\eta_{mimo} = k_1(l_s, b, \tilde{P_d}, \tilde{P_f}, \overline{P_{eth}}, M_t, N_r)(l_s - M_t b)(1 - \overline{P_{eth}})^{M_t l_s} \quad (k_1(l_s, b, \tilde{P_d}, \tilde{P_f}, \overline{P_{eth}}, M_t, N_r) + k_{tot})l_s + E_{tot}(\tilde{P_d}, \tilde{P_f})
\]
(66)

Finally, taking into account a fixed average BER threshold \(\overline{P_c} = \overline{P_{eth}}\) expressing the energy consumption per bit \(k_1\) as a function of \(P_{eth}\), the simplified optimization problem boils down to
\[
\begin{align*}
\min_{b} & \quad \max_{l_s} \eta_{\text{mimo}}(l_s, b, \tilde{P}_d, \tilde{P}_f, \bar{P}_{\text{eth}}, M_t, N_r) \quad (72a) \\
\text{subject to} & \quad c_1^{\text{mimo}}(l_s, b, \tilde{P}_d, \tilde{P}_f, M_t) \leq 0 \quad (72b) \\
& \quad c_2^{\text{mimo}}(l_s, b, \tilde{P}_d, \tilde{P}_f, M_t, N_r) \leq 0 \quad (72c) \\
& \quad c_3^{\text{mimo}}(l_s, b, P_d, \tilde{P}_f, \bar{P}_{\text{eth}}) \leq 0 \quad (72d) \\
100 < l_s < 1000, b \in \{2, 3, 4, ..., 10\}, \quad (72e)
\end{align*}
\]
where both \(b \) and \(l_s \) are discrete integers.

VII. ALGORITHMS TO DETERMINE THE OPTIMAL PACKET SIZE

A. Exhaustive Search Based Algorithm-1

An exhaustive search based Algorithm-1 is proposed to solve the formulated optimization problem (72a) to (72e). As both modulation level (\(b\)) and packet size (\(l_s\)) are discrete integers, the energy consumption per bit \(k_1\) and all the other constraints \(c_1^{\text{mimo}}, c_2^{\text{mimo}}\) and \(c_3^{\text{mimo}}\) related to the interference duration, end-to-end delay and peak transmit power constraints are calculated based on (72b), (72c) and (72d) for all the possible combinations of modulation levels (\(b\)) and packet length \(l_s^*\) obtained which maximize the cost function (72a). The value of the cost function at \(l_s^*\) is denoted as \(\eta_{\text{mimo}}^*\). Same process is repeated for a range of modulation levels (\(b\)) supported by the variable rate m-QAM modulation scheme where \(b\) ranges from \(b = \{2, 3, 4, ..., 9\}\). Once the values of the maximized cost function for each \(b\) is obtained, the minimum \(\eta_{\text{mimo}}^*\) is selected and its corresponding \(l_s^*\) and \(b\) provides the optimal packet size and modulation level.

B. Conventional Karush-Kuhn-Tucker (KKT) based algorithm

An efficient suboptimal algorithm based on solving the Karush-Kuhn-Tucker conditions denoted as Algorithm-2 is proposed to solve the optimization problem. In Algorithm-2 the packet size \(l_s\) is considered to be continuous unlike Algorithm where it was considered to be discrete in nature. For the next step, the cost function in this technique is modified to a new Lagrangian function which comprises of the original cost functions and the constraint functions which is now multiplied by the Lagrangian multipliers \(\lambda_1, \lambda_2\) and \(\lambda_3\) corresponding to the three constraint functions (72b), (72c) and (72d). The root of derivative of this newly formed Lagrangian function with respect to the decision variable \(l_s\) needs to be calculated. Furthermore, along with the Lagrangian function, the feasibility of the constraint functions, the complimentary slackness condition related to the Lagrangian variable and the constraints and the negativity of the Lagrangian conditions needs to be satisfied which provided what is known as the feasible KKT point. Negativity condition of the Lagrangian multipliers is because in our case for each modulation level, the original cost function is a maximization problem.

For each of the modulation level (\(b\)), the KKT point and its corresponding value of the original cost function (\(\eta_{\text{mimo}}\)) is found. Finally, just like Algorithm-1 the value of the KKT point corresponding to the minimum (\(\eta_{\text{mimo}}\)) or \(k_1\) value provides optimal packet size \(L_{\text{opt}}\).

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Algorithm-1 Exhaustive Search to determine OPS for MIMO-CSRNs.

1. **Initialize:** \(L_s = \{100, 101, ..., 1000\}, \quad b = \{2, 3, 4, ..., 9\}, \quad P_d = 0.9, \quad P_f = 0.1, \quad \bar{P}_{\text{eth}}\).
2. **for** \(i=1\) **to** \(max(b)\) **do**
3. **for** \(j=1\) **to** \(max(l_s)\) **do**
4. **Calculate:** \(\eta_{\text{mimo}}(l_s(j), b(i), \tilde{P}_d, \tilde{P}_f, \bar{P}_{\text{eth}}, M_t)\) from (72a), \(c_1^{\text{mimo}}(l_s(j), b(i), \tilde{P}_d, \tilde{P}_f, \bar{P}_{\text{eth}}, M_t)\) from (72b), \(c_2^{\text{mimo}}(l_s(j), b(i), \tilde{P}_d, \tilde{P}_f, \bar{P}_{\text{eth}}, M_t)\) from (72c) and \(c_3^{\text{mimo}}(l_s(j), b(i), P_d, \tilde{P}_f, \bar{P}_{\text{eth}})\) from (72d)
5. **end for**
6. **Find:** Set of \(l_s\) values \((\hat{l}_s^i)\) for which both \(c_1^{\text{mimo}}(l_s) \leq 0\) and \(c_2^{\text{mimo}}(l_s) \leq 0\) \(\forall b = b(i)\)
7. **if** \(L_s\) **then**
8. **Find:** \(\eta_{\text{mimo}}^* = max(\eta_{\text{mimo}})\) at \(b=b(i)\)
9. **if** \(\eta_{\text{mimo}}(l_s^*, b(i)) \leq 0\) **then**
10. \(l_s^* = l_s^*\)
11. **else if** \(c_3^{\text{mimo}}(l_s^*, b(i)) > 0\) **then**
12. **Find:** \(\eta_{\text{mimo}}(l_s^*, b(i)) = \min(\eta_{\text{mimo}})\)
13. **end if**
16. **Find:** \(L_{\text{opt}}^* = \{l_s^*\} \subset L_s\) and \(\min(\eta_{\text{mimo}}) \leq \eta_{\text{mimo}}(l_s^*, b(i))\)
17. **Find:** \(\eta_{\text{mimo}}(l_s^*, b(i)) = \eta_{\text{mimo}}^*\)
18. **end if**
20. **else**
21. **Find:** \(\eta_{\text{mimo}}^* = \min(\eta_{\text{mimo}})\)
25. **end for**
26. **Find:** \(b_{\text{opt}}^* = \{b_{\text{opt}}^*\} \subset b\)
27. **end if**

\[
\frac{\partial L(l_s, \lambda_1, \lambda_2, \lambda_3)}{\partial l_s} = \eta_{\text{mimo}}(l_s) + \lambda_1 \frac{\partial c_1^{\text{mimo}}(l_s)}{\partial l_s} + \lambda_3 \frac{\partial c_3^{\text{mimo}}(l_s)}{\partial l_s} = 0
\]

\[
\begin{cases}
c_1^{\text{mimo}}(l_s) \leq 0 \\
c_2^{\text{mimo}}(l_s) \leq 0 \\
c_3^{\text{mimo}}(l_s) \leq 0 \\
\lambda_1 c_1^{\text{mimo}}(l_s) = 0 \\
\lambda_2 c_2^{\text{mimo}}(l_s) = 0 \\
\lambda_3 c_3^{\text{mimo}}(l_s) = 0
\end{cases}
\]

Fig. 3 shows the schematic representation of Algorithm-2 with different subalgorithms illustrated in the appendix.
Table I. SIMULATION PARAMETERS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>Channel Bandwidth</td>
<td>1 MHz</td>
</tr>
<tr>
<td>$f$</td>
<td>Operating Frequency</td>
<td>2.4 GHz</td>
</tr>
<tr>
<td>$P_{f,om}$</td>
<td>Probability of primary occupancy</td>
<td>$\frac{3}{4}$</td>
</tr>
<tr>
<td>$t_{oa}$</td>
<td>Avg. channel idle time</td>
<td>200 ms</td>
</tr>
<tr>
<td>$d_{cs}$</td>
<td>Distance between two cognitive secondary users</td>
<td>10 m</td>
</tr>
<tr>
<td>$n_i$</td>
<td>Uncoded symbol rate</td>
<td>10 bauds</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Path-loss exponent</td>
<td>2.4-3.5</td>
</tr>
<tr>
<td>$G_t(G_r)$</td>
<td>Gain of the transmitter and receiver of secondary users</td>
<td>5 dB</td>
</tr>
<tr>
<td>$M_f$ and $N_f$</td>
<td>Link margin and noise figure</td>
<td>5 dB and 10 dB</td>
</tr>
<tr>
<td>$P_{Tx}$</td>
<td>Power consumed by the transmitter circuit</td>
<td>98.5 mw</td>
</tr>
<tr>
<td>$P_{Rx}$</td>
<td>Power consumed by the receiver circuit</td>
<td>125 mw</td>
</tr>
<tr>
<td>$N_0$</td>
<td>One sided thermal noise</td>
<td>-171 dBm/Hz</td>
</tr>
<tr>
<td>$l_{sens}$</td>
<td>Size of the header in bytes</td>
<td>6 bytes</td>
</tr>
<tr>
<td>$P_{sens}$</td>
<td>Power consumed by the circuit due to channel sensing</td>
<td>110 mw</td>
</tr>
<tr>
<td>$E_{R,off}$</td>
<td>energy consumed during handoff</td>
<td>2 mJ</td>
</tr>
<tr>
<td>$E_{rec}$</td>
<td>Local energy consumption per bit at $T_e$</td>
<td>53.2 mJ</td>
</tr>
<tr>
<td>$N_c$</td>
<td>No. of receiver antennas or receivers at the intermediate $R_c$ cluster</td>
<td>1</td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>Gain of the receiver of secondary users</td>
<td>$\frac{3}{4}$</td>
</tr>
<tr>
<td>$V$</td>
<td>Po</td>
<td>5</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Channel link</td>
<td>2.98 m</td>
</tr>
<tr>
<td>$h$</td>
<td>Header of the header in bytes</td>
<td>6 bytes</td>
</tr>
<tr>
<td>$P_{sens}$</td>
<td>Power consumed by the circuit due to channel sensing</td>
<td>110 mw</td>
</tr>
<tr>
<td>$E_{R,off}$</td>
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</tbody>
</table>

![Fig. 3. Newton-Raphson technique assisted KKT-based Algorithm-2 [11]](image)
scenario for P2P-CRSN system, OPS value at M=6 turns out to be 318 bits while at M=20 it will be 301 bits. This is due to the fact that with the increase in the number of users in the system results in increase of the energy consumption per bit k1 both for the MIMO and P2P CRSN system alike (70). Since it is already proven that the cost function \( \eta \) will be an increasing function of k1. For a fixed value of \( I_s \), the value of the function is the energy-efficiency reliability metric increases and the optimal \( I_s \) for which the cost function gets maximized reduces with increasing k1. This leads to a sharp decline in the OPS value or P2P-CRSN architecture but for MIMO-CRSN system, the cost function \( \eta_{\text{mimo}} \) (72a) apart from the k1 component also contains the overhead energy consumption at the transmitter end due to local intracluster information exchange to enable Alamouti encoding. This significant overhead energy consumption component in the denominator of the cost function scales down the cost function and the rate of decrease in the OPS value with increasing k1 or M is not as high as compared to P2P-CRSN system. Similar trend could be observed when the network conditions are made severe with increased interference level \( SNR_{pr} = -5 \) dB. However OPS value for P2P-CRSN could be obtained only upto 6 users beyond which the peak power constraint of 20 dBm for each sensor node could not be satisfied to meet the average BER threshold \( P_{\text{eth}} = 10^{-3} \). In case of MIMO-CRSN since the transmit power is divided equally among the two \( M=2 \) sensor nodes, optimal packer size could easily obtained upto 20 users in system with OPS values ranging from 287 to 284 bits at \( M=2 \) and \( M=20 \) users.

Fig. 4(b), shows the OPS value for the MIMO-CRSN and P2P-CRSN architecture for increasing number of cognitive users in the system for different number of available channels \( C \) for a fixed interference power \( SNR_{pr} = -10 \) dB. In case of \( (2 \times 1) \) MIMO-CRSN system as the number of available channels is reduced from \( C=30 \) to \( C = 10 \), a marginal reducing in the OPS value could be observed. Whereas in P2P-CRSN the reduction in the OPS value for reducing the number of channels from 30 to 10 is quite significant. At \( M=10 \), the OPS value when \( C=30 \) is around 310 bits and OPS value is 297 bits when \( C=10 \). The reason is contributed by the fact that decrease in the number of channels leads to increased Co-selection probability (4) as chances of more than single cognitive user selecting the same channel for data transmission increases. This leads to increases \( k_1 \) to attain a specific BER threshold thus reducing the optimal packet size length. However, just like Fig. 4(a), the reason for marginal decrease in the OPS value for MIMO-CRSN is due to the intracluster local energy consumption overhead. Even though marginal but the reason for decrease in the OPS value for decrease in channel numbers for MIMO-CRSN is same to that of point-to point CRSN. Moreover, again in this figure it could be observed that the results obtained from Algorithm-1
and Algorithm-2 are perfect match.

The OPS values for different intracluster communication energy requirement \( k_{loc} \) is analyzed in Fig. 4(c). The motivation for such analysis is due to the varied mean intracluster distance among the sensor nodes. Higher intracluster distance value leads to higher local energy consumption per bit \( k_{loc} = 5 \mu J \) and for shorter mean distance it could be as low as 2 \( \mu J \). Two different values of \( k_{loc} \) fixed at 5 \( \mu J \) and 2 \( \mu J \) are considered based on which the OPS value are estimated at \( SNR_{pr} = -10 \) dB and \( SNR_{pr} = -5 \) dB respectively. It could be observed that when \( k_{loc} \) is 2 \( \mu J \), the OPS values are higher than that of \( k_{loc} = 5 \mu J \) both at \( SNR_{pr} = -10 \) dB and -5 dB. This is due to the fact \( k_{loc} << k_1 \) and it is absolutely independent of the long haul energy consumption per bit \( k_1 \); as it is assumed that communication for local intracluster information exchange takes place among channels which is not accessed by the non-cognitive users. As the \( k_{loc} \) value gets higher for a fixed \( k_1 \) and \( k_p \), the nature of the overall cost function \( \eta_{p,mimo} \) is dominated by the \( k_{loc} \). As the \( k_{loc} \) gets higher from 2 \( \mu J \) to 5 \( \mu J \), the value of the cost function for each packet length scales down. However, in this case as the cost function reduces, the optimal packet length \( l_p \) which would maximize the cost function would decrease unlike the previous results in Fig. 4(a) and Fig. 4(b) where increasing cost function value with increasing \( k_1 \) would lower the optimal packet size value.

In Fig. 4(d), the OPS value for different number of cognitive users are shown for two different values of maximum interference duration to non-cognitive users threshold \( I_{max} \) at 5\% and 4\% respectively. Comparison is made both (2×1) MIMO-CRSN system and well as point-to-point CRSN system. It could be observed that for both the \( SNR_{pr} \) values at -10 dB and -5 dB, there is a sharp decline in the OPS values when the interference constraint duration constraint that is \( c_{\text{min}} \) is made active and more severe. If we compare the results of Fig. 4(d) with Fig. 4(a), when the interference constraint is inactive as in Fig. 4(a), for the MIMO-CRSN system the OPS values at \( SNR_{pr} = -10 \) dB ranges from 292 to 287 while for \( SNR_{pr} = -5 \) dB it varies from 287 to 285 when \( M \) is varying between 2 to 20 users. Under the influence of the constraint, this results sharply falls to 250 to 200 bits when for \( SNR_{pr} = -5 \) dB when \( I_{max} = 4\% \) and 287 to 248 bits at \( SNR_{pr} = -10 \) dB at \( I_{max} \) equal to 4\%. Futhermore, when compared with point-to-point CRSN system, OPS could be determined only upto 12 users at \( SNR_{pr} = -10 \) dB and only 2 users at \( SNR_{pr} = -5 \) dB with lower values at around 150 bits and 100 bits. This is caused because of the fact that in MIMO-CRNS system, the total number of users actually contending over \( C \) data channels is scaled down by the number of users present in each MIMO cluster (\( M_j \)) as these \( M_j \) users in each cluster will transmit over the same channel. Thus the contention is reduced in MIMO-CRNS system.

Fig. 4(e) and Fig. 4(f) the OPS values are shown for varying delay sharpening factor \( s_f \) at \( SNR_{pr} = 10 \) dB and -5 dB. As total end to end delay \( \tau_d = \frac{s_f M K}{R_s} \) where \( R_s \) is the fixed symbol rate, reducing \( s_f \) leads to reduction of the time duration \( \tau_d \) thus making the delay constraint more severe. It could be observed that when \( s_f \) is reduced from 1 to 0.8 at \( SNR_{pr} = 10 \) dB, in case of (2×1) MIMO-CRNS scenario, the OPS value remains same upto 6 users both for \( s_f = 1 \) and \( s_f = 0.8 \) around 290 bits but as number of cognitive users (\( M \)) in the system increases, the OPS value for \( s_f = 0.8 \) increases to 318 bits and saturates. This is because as long a \( s_f = 1 \) and \( M=6 \), the delay constraint is not active and the results obtained is similar to that of OPS values in Fig. 4(a). However, as \( s_f \) is reduced to 0.8 the delay constraint becomes active for users above 6. This leads to higher packet size as in this case the optimal packet size will be determined by the constraint function and not necessarily the packet size which would maximized the cost function as in other cases described. When this result is compared with point-to-point CRSN architecture, it could be also observed that under \( s_f = 0.8 \), OPS could be obtained only upto 12 users in the system as to MIMO-CRNS which enables to determine OPS for 20 users. This is because MIMO-CRNS selects higher modulation level as compared to P2P-CRNS. Furthermore, in this case too the results obtained from exhaustive search based Algorithm-1 and KKT based Algorithm-2 provides a perfect match which proves the mathematical accuracy of our proposed algorithms under active constraint cases. For Fig. 4(f), as the \( SNR_{pr} \) is made more severe to -5 dB, the trends of the results remains same with only difference being now at \( s_f = 0.8 \), the OPS saturates at 294 bits when \( M=2 \).

![Fig. 5. a. Elapsed CPU time versus no.of cognitive users (M) for Algorithm-1 and 2 b. OPS versus the average busy time at M=6, SNR_{pr} = -5 dB for different values of v_p, c. OPS versus channel sensing time at M=6, SNR_{pr} = -5/-10 dB for different values of T_s, d. OPS versus varying distance for different distributed and centralize channel access schemes at SNR_{pr} = -5 dB, M=12.](image-url)
algorithms based on exhaustive search (Algorithm-1) and Newton-Raphson numerical technique assisted conventional KKT based (Algorithm-2) are heuristic in nature and a straight forward analysis of its order of complexity is trivial. However, a rather indirect approach to estimate the performance of the proposed algorithms is to use the MATLAB command CPUTIME which shows a rough estimate of the time consumed by the CPU during simulation to execute Algorithm-1 and Algorithm-2 for a given number of users in the system. Although, this estimation technique depends upon numerous factors like the type of computer processor used during the simulation (Intel i5 in our case) etc. however, the mentioned approach helps us to evaluate and have a holistic overview of the performance of the proposed algorithms. The elapsed CPU-time is this case is estimated for 10000 simulation seeds and then the average is calculated. It could be observed that Algorithm-2 takes 4 ms to obtain the optimum packet size while for Algorithm-1 the time taken is in the order of 680 ms. This is contributed by the fact that for exhaustive search technique, the search space spans over all the discrete values of packet length that is \( l_p = \{100, 101, \ldots, 2000\} \) and modulation level \( b = \{2, 3, 4, 5\} \). The cost function and the constraint functions derived from (72a) to (72d) are calculated for all the \( l_p \) and \( b \) values discretely to check the feasibility and determination of the OPS. The elapsed time duration could be shortened though by reducing the search space upon the application but however in case of Algorithm-2, the search calculation depends upon the number of iterations of the Newton-Raphson Algorithm to find the root of the lagrangian based modified cost function (83) to determine the OPS. Furthermore, to determine the feasibility considering the constraint functions, the lagrangian variables of the formulated KKT solution and mathematically simplified constraint functions as shown in (88) to (95), helps to determine the feasible optimal point without discretely checking each and every packet length to determine the OPS.

In Fig. 5(b), the OPS value is shown for varying average busy time \( l_p \) and average idle time \( v_p \) of the non-cognitive users for MIMO-CRSN system for a given number of users in the system \( M=6 \). With increase in \( l_p \) from 100 to 300 ms for average idle time \( v_p \) fixed at 100, 200 and 300 ms, the packet size increases. This is because as \( l_p \) is increasing, the probability of occupancy \( P_{on} \) of the non-cognitive users increases. With increasing \( P_{on} \), the energy consumed due to channel switching increases significantly which scales down the cost function and the size which maximizes the cost function \( \eta_{\text{mimo}} \) increases. This counterintuitive result hold true even for point to point CRSN system as shown in our previous work [11]. Similarly as \( v_p \) increases from 100 ms to 300 ms, the \( P_{on} \) decreases since \( P_{on} = \frac{l_p}{l_p+v_p} \). Therefore, for a fixed \( l_p \), the OPS value will be maximum when \( v_p=100 \) ms. Results ousing Algorithm-1 and Algorithm-2 shows perfect point without discretely checking each and every packet length to determine the OPS.

In Fig. 5(c), the OPS value when \( SNR_{pr} = -10 \) dB. This trend is different than the results shown in [11]. The reason for decrease in OPS value with increasing sensing time is due to the fact that at higher channel SNR at around -10 dB, the channel sensing time is of the order of micro seconds which is very low. However, as channel sensing time \( t_s \) increases the probability of false alarm \( P_f \) would decrease monotonically for a fixed \( P_d \) value. This decrease in the probability of false alarm results into decrease in the overhead energy consumed due to channel handoff \( (E_{h_f}) \) as probability of channel switching \( (P_{sw}) \) decreases. This leads to an increase in the cost function \( \eta_{\text{mimo}} \) as \( E_{h_f} \) is in the denominator of the cost function. Therefore, the packet size now at which the cost function will be maximized will reduce as \( t_s \) would increase which leads to lower packet size. When the \( SNR_{pr} \) is made more severe at -5 dB, the OPS value will saturate beyond 120 \( \mu \)s. This is because with increased value of \( SNR_{pr} = -5 \) dB, the effect of increasing sensing time will have minimum impact on the probability of false alarm. Therefore, the OPS value will not reduce any further beyond a specific \( t_s \). In Fig. 5(d), the behaviour of the OPS with varying transmission distance is analyzed for two different access schemes based on MIMO distributed time slotted medium access control denoted as MIMO-TS-CMAC and a CSMA/CA assisted centralized common control channel based medium access control denoted as MIMO-CC-CMAC and the same is compared with point-to-point CRSN system with distributed and centralized access schemes. Under severe network conditions when \( SNR_{pr} = -5 \) dB and the number of cognitive users in the system \( M=12 \), it could be observed that distributed MIMO-CRSN system outperforms the single point-to-point distributed CRSN system as OPS could be obtained only upto 8 m in a point to point scenario. However, the packet size for both the cases would reduce as the transmission distance increases since energy consumption per bit \( k_1 \) would increase with increasing distance and the OPS would naturally decrease. The OPS value for P2P-DTS-CMAC will be lesser than that of P2P-CC-CMAC as in the centralized system the probability of co-user selection does not exist which negates the risk of more than one cognitive user selecting the same data channel. This reduces the \( k_1 \) value for the centralized system thus providing higher OPS values. Same arguments holds true for the MIMO-CRSN scenario however, this OPS gap will not be significant due to additional overhead energy consumed due to local information exchange as \( k_{loc} \) is fixed at 5 \( \mu \)J.

In Fig. 6(a) and Fig. 6(b), the overall energy consumption of the system is shown for a \((2 \times 1)\) MIMO based distributed time slotted cognitive channel access scheme (MIMO-DTS-CMAC) for varying transmission distance at \( SNR_{pr} = 10 \) dB and \( SNR_{pr} = -5 \) dB. When \( SNR_{pr} = -10 \) dB. Moreover, the results are compared with cognitive point-to-point CRSN system with distributed access scheme. In Fig. 6(a), the overall energy consumption is shown for two test cases when number of cognitive users in the system \( M=6 \) and \( M=12 \). It could be observed that for increasing transmission distance from 2 to 20 m the overall energy consumption will increase both for MIMO and P2P CRSN framework which is quite obvious. However, the MIMO-DTS-CMAC with optimal packet size
The similar trend holds true when $M$ is increased to 12 users. Another interesting observation is P2P-CRSN system could support optimal packet size only up to $d_{ss} = 14$ m above which feasible optimal packet length could not be obtained due to the transmit power constraint. Fig. 6(b) shows similar results to that of Fig. 6(a) with only difference being the critical distance beyond which MIMO-DTS-CMAC with OPS outperforms P2P-CRSN system with OPS is further reduced to 8 m and 6 m when $M = 6$ and 12 respectively. For Fig. 6(c) and Fig. 6(d), a CSMA/CA dependent $(2 \times 1)$ MIMO based centralized common control channel assisted cognitive medium access control (MIMO-CC-CMAC) is considered [39]. The trends of the results are similar to that of the last two figures but however, the critical distance beyond which the MIMO-CC-CMAC outperforms the P2P-CC-CMAC system beyond 35 m both for $M = 6$ and $M = 12$ at $SNR_{pr} = -10$ dB while at $SNR_{pr} = -5$ dB the critical distance would be 20 m beyond which OPS could not be obtained for the centralized point-to-point system. Furthermore, if we carefully examine the results obtained from the distributed system and compare it with the centralized MIMO based access scheme, the overall energy consumption will be lesser for the centralized system as multiple cognitive users does not selects the same channel. In Fig. 6(e), the overall end to end delay analysis is shown for the two aforementioned channel access schemes both for the proposed MIMO-CRSN system and point-to-point system with OPS. It could be observed that in terms of overall delay for point to point system with optimal packet size will be lower than any of the MIMO based CRSN system up to a critical distance of 37 m for the centralized system and 13 m for the distributed system beyond which delay for the P2P mode will increase and as further no feasible optimal packet size could be obtained. This is mainly due to the overhead delay caused due to the local information exchange at the transmitter (44)
and non-feasibility of OPS beyond a certain distance threshold as explained in Fig. 5(d). Furthermore, the MIMO-CC-CMAC incurs more delay due to the overhead delay due to the negotiation over the common control channel \( T_{\text{cn}} \), due to the DIFS, SIFS timing interval while transmitting the RTS/CTS packets over the common control channel before the cognitive transmitter and receiver agrees on a particular channel with an assumption that a node will try at least 5 retransmissions over the control channel before declaring it to be busy and back off for a given duration of time. Fig. 6(f) illustrates the overall interference duration caused to the non-cognitive users as compared to the MIMO-DTS-CMAC and other point to point access schemes both with MIMO and point-to-point mode of transmission. The MIMO-CC-CMAC will cause minimum interference duration to the non-cognitive users as compared to MIMO-DTS-CMAC and other point to point access schemes because as shown in Fig. 5(d), the OPS value for MIMO-CC-CMAC will be minimum as compared to MIMO-DTS-CMAC and the two other point to point access schemes with OPS. Since the average interference duration is dependent on the optimal packet size from (38) therefore, the interference duration for MIMO-CC-CMAC will be minimum.

IX. CONCLUSION AND FUTURE WORKS

This paper proposed a novel optimal packet size determination framework for MIMO-CRNs. An optimization model is framed to determine the OPS which apart from determining the OPS, guarantees the minimum energy consumption. Two key algorithms are proposed to evaluate the OPS. From the simulation results it could be seen that the elapsed CPU time for the KKT based Algorithm-2 outperforms Algorithm-1 by a significant margin. The CPU elapsed time for Algorithm-2 is of the order of 5 to 10 ms while for Algorithm-1 it is 600 ms. Although this comparative analysis is shown using MATLAB tool but it is evident from the simulations that then Algorithm-2 would consume much lower execution time and will be a feasible option for real time implementation. Our algorithms is introduced to a centralized common control channel based strategy to compare its performance with the distributed one. In normal scenario, the CSMA/CA assisted centralized common control channel based cognitive access scheme (MIMO-CC-CMAC) with OPS will outperform the distributed (MIMO-DTS-CMAC) with OPS in terms of overall energy consumption. In terms of delay, under unconstrained conditions, MIMO-CC-CMAC will incur additional delay as compared to both MIMO and P2P due to the intracluser overhead. In terms of overall interference duration caused to the non-cognitive users, MIMO-CC-CMAC with OPS causes minimum interference to the non-cognitive users as compared to the MIMO-DTS-CMAC scheme with OPS and other point-to-point schemes. In future more efficient access strategies could be couple with our proposed MIMO-CRNS architecture. Energy harvesting MIMO-CRNS framework with OPS would be an interesting area to which our work could be further extended.

APPENDIX A

FOR SUBALGORITHMS- 2.1, 2.2, 2.4 AND 2.4

A. Subalgorithm-2.1

\[
\frac{\partial \eta}{\partial l_s} = \frac{1 - P_{\text{eth}}}{(k_1 + k_{\text{loc}}) l_s} \left\{ M_k k_1 (l_s - M_i h) \log (1 - P_{\text{eth}}) + \frac{Z_5 (l_s)}{(k_1 + k_{\text{loc}}) l_s + Z_3} \right\},
\]

(75)

where

\[
Z_5 (l_s) = k_1 (k_{\text{loc}} l_s^2 + Z_3 l_s - k_{\text{loc}} M_l h l_s) + Z_3 k_1 (l_s - M_i h k_1) + k_1 (k_{\text{loc}} M_l h + k_1 M_i h)
\]

(76)

The derivative of \( k_1 \) calculated to be as

\[
k_1 = (1 + \alpha) P_{\text{out}}
\]

(77)

\[
k_1' = (1 + \alpha) p_l N_0 \gamma_a
\]

(78)

where \( p_l = (\frac{4n^2}{4^2 + N_0}) \) is the pathloss component.

\[
\gamma_a = \frac{1}{L} \left\{ K_a \frac{1}{P_{\text{eth}} (1 + \Omega (a - 1))} \right\} \left( a - 1 \right) \frac{K_a}{P_{\text{eth}}} \Omega' (l_s),
\]

(79)

where

\[
a = \left( \frac{N_0 + P_{\text{pp}}}{N_0} \right)^L
\]

(80)

\[
K_a = \frac{4}{b} \left( 1 - \frac{1}{2^7} \right) \left( \frac{2L - 1}{L} \right) \frac{1}{2^L} \left( \frac{3b}{(2^6 - 1) M_l} \right)^{-L}
\]

(81)

\[
\Omega' (l_s) = P_{\text{on}} P_{\text{off}} \left( 1 - \tilde{P}_f \right) P_{\text{c}} \frac{1}{1 - P_{\text{c}}} e^{\frac{l_s}{2L}} - \left( 1 - \tilde{P}_d \right) \frac{1}{1 - P_{\text{c}}} e^{\frac{l_s}{2L}}
\]

(82)

Based on (75) to (82), the modified cost function \( f(l_s) \) to calculate optimal packet size \( l_s^{\text{opt}} \) turns out to be

\[
f(l_s) = \left\{ M_k k_1 (l_s - M_i h) \log (1 - P_{\text{eth}}) + \frac{Z_5 (l_s)}{(k_1 + k_{\text{loc}}) l_s + Z_3} \right\}
\]

(83)

\[
\gamma_a'' = K_a (1 + (a - 1) \Omega (l_s)) \left( a - 1 \right) \Omega'' (l_s) + K_a \left[ \Omega' (l_s) \right]^2 \left( \frac{1}{L} - 1 \right) \left( 1 + (a - 1) \Omega (l_s) \right)^2 (a - 1)
\]

(84)
\[ Z_2'(l_3) = kloc k_3'' I_3' + 2 k_1' kloc l_3 - k_3' I_3 + k_3' + 2 M_h k_1 k_3' + M_h kIoc k_3' + Z_3(k_3' l_3 + 2 k_1' - M_h k_3') \]  
\hspace{1cm} (85)

\[ f''(t_a) = 2 \log(1 - \frac{1}{R_{eh}}) + \left\{ \begin{array}{l} (k_1 + k_{loc}) l_3 + Z_3 \left( -2 k_3' l_3 + k_1 + k_{loc} \right) Z_5(l_3) \\
\{(k_1 + k_{loc}) l_3 + Z_3\}^2 \end{array} \right. \]  
\hspace{1cm} (86)

\[ \Omega''(t_a) = \frac{P_{on} P_{off} \left( \frac{1}{(R_{p} + \mu) + \mu} - \left( \frac{1}{P_f} \right) P_{off} + \frac{1}{(R_{of} + \mu) + \mu} \right)}{P_{off} (1 - P_f) + P_{on} (1 - P_d)} \]  
\hspace{1cm} (87)

\section{Subalgorithm-2.2}

\[ c_1^{mimo'}(l_3, b) = A_1 + l_3 B_1, \]  
\hspace{1cm} (88)

where

\[ A_1 = \frac{P_{on} + P_{f} P_{off} \frac{1}{R_{p}} e^{-\frac{1}{R_{p}}} + \mu}{P_f + P_{off}^{\mu}} \]  
\hspace{1cm} (89)

\[ B_1 = \frac{1}{(P_{f} + P_{off}^{\mu})^2} \left( (P_{f} + P_{off}^{\mu}) \right) \]  
\hspace{1cm} (90)

\[ P_{2}^{\mu} = \frac{1}{R_{p}} e^{-\frac{1}{R_{p}}} + \frac{1}{R_{of}} \]  
\hspace{1cm} (91)

\section{Subalgorithm-2.3}

\[ c_2^{mimo'}(l_3, b) = 2 \frac{M_h P_{eh}^{\mu}}{b} l_3 + \left( \frac{1}{b} + \frac{1}{b_{loc}} + R_s M_h \frac{P_{eh}^{\mu}}{R_{p} + \mu} \right) R_s M_h \frac{P_{eh}^{\mu}}{R_{p} + \mu} \]  
\hspace{1cm} (92)

where

\[ A = \frac{M_h P_{eh}^{\mu}}{b}, B = \left( \frac{1}{b} + \frac{1}{b_{loc}} + R_s M_h \frac{P_{eh}^{\mu}}{R_{p} + \mu} \right) \]  
\hspace{1cm} (93)

\[ C_{a} = \left( M_h R_s + \tau_{tot} + R_s - M_h \right) \]  
\hspace{1cm} (94)

\section{Subalgorithm-2.4}

\[ c_3^{mimo'}(l_3, b) = \frac{N_{3} P_{eh}^{\mu}}{M_h L} \left[ K_x P_{eh}^{\mu} \{1 + \Omega(a - 1)\} \right]^{a - 1} \left( a - 1 \right) \frac{K_x}{P_{eh}^{\mu}} \Omega'(l_3), \]  
\hspace{1cm} (95)

---

**Subalgorithm-2.1 Both delay and interference duration constraints inactive**

**Require:** \( P_{on}, c_1^{mimo'}(l_3), c_2^{mimo'}(l_3), c_3^{mimo'}(l_3) \)

1: Initialize: \( l_3 = 0, \theta_3 = 0, P_{f} = P_{d} = P_{off} = 0.9, P_f = 0.1 & P_{off}^{\mu} = 10^{-3} \)

2: while \( \theta_3 < i \)

3: Calculate: \( f''(t_a) \) & \( f''(t_a) \) Using (83) & (86)

4: if \( \theta_3 - \theta_3 < 10^{-6} \)

5: Calculate: \( f''(t_a) \) & \( f''(t_a) \)

6: return \( \theta_3, c_1^{mimo'}(l_3), c_2^{mimo'}(l_3), c_3^{mimo'}(l_3) \)

7: else

8: \( \theta_3 + 1 \)

9: end if

10: end while

---

**Subalgorithm-2.3 Interference duration constraint inactive and delay constraint active**

**Require:** \( P_{on}, c_1^{mimo'}(l_3), c_2^{mimo'}(l_3) \)

1: Initialize: \( P_{f} = P_{d} = 0.9, P_f = 0.1 & P_{off}^{\mu} = 10^{-3} \)

2: Calculate: \( P_{on}^{\mu} = P_{f} + \frac{P_{off}^{\mu}}{R_{p} + \mu} \)

Using values of \( A, B \) & \( C_{a} \) from (92) to (94).

3: if \( P_{on}^{\mu} \) & \( P_{off}^{\mu} \) then

4: if \( c_1^{mimo'}(l_3) \) then

5: \( P_{on}^{\mu} = \frac{P_{on}^{\mu}}{R_{p} + \mu} \)

6: else

7: \( P_{on}^{\mu} = \frac{P_{on}^{\mu}}{R_{p} + \mu} \)

8: end if

9: if \( P_{on}^{\mu} \) & \( P_{off}^{\mu} \) then

10: if \( c_1^{mimo'}(l_3) \) then

11: \( P_{on}^{\mu} = \frac{P_{on}^{\mu}}{R_{p} + \mu} \)

12: Calculate: \( c_1^{mimo'}(l_3) \)

13: else

14: return \( c_1^{mimo'}(l_3) \) & \( K_{on}^{\mu} \)

15: if \( K_{on}^{\mu} < 0.1 \) \( c_1^{mimo'}(l_3) \) then

16: \( P_{on}^{\mu} = \frac{P_{on}^{\mu}}{R_{p} + \mu} \)

17: Calculate: \( c_1^{mimo'}(l_3) \)

18: else

19: return \( c_1^{mimo'}(l_3) \) & \( K_{on}^{\mu} \)

20: end if

21: \( P_{on}^{\mu} = 0, c_1^{mimo'}(l_3) = 0 \) \& \( K_{on}^{\mu} = 0 \)

22: end if

23: end if

24: \( P_{on}^{\mu} = 0, c_1^{mimo'}(l_3) = 0 \) \& \( K_{on}^{\mu} = 0 \)

25: end if
Subalgorithm-2.2 Interference duration constraint active and delay constraint inactive

Require: $\tau_{\text{opt}}$, $c_1(\tau_{\text{opt}})$ & $k_{\text{opt}}$

1: Initialize: $i^e = 100$, $i = 1$, $\text{iter} = 100$, $P_D = \hat{P}_D = 0.9$
2: while $i < \text{iter}$ do
3: \[ i^e = i^e + 1 \]
4: Calculate: $c_1^{\text{mimo}}(\tau_{\text{opt}})$ & $c_1^{\text{mimo}}(\tau_{\text{opt}})$ Using (40) and (88) to (91)
5: if $|\tau_{\text{sum}} - \tau_{\text{opt}}| \leq 10^{-3}$ then
6: \[ i^e = \tau_{\text{opt}} \]
7: Calculate: $\eta(\tau_{\text{opt}})$, $c_2^{\text{mimo}}(\tau_{\text{opt}})$, $c_2^{\text{mimo}}(\tau_{\text{opt}})$ & $k_{\text{opt}}$
8: Using (75) to (82), (46), (47), (67) and (70)
9: \[ \lambda_1 = \frac{n(\tau_{\text{opt}})}{\tau_{\text{opt}}} \]
10: if $\lambda_1 < 0$ & $c_2^{\text{mimo}}(\tau_{\text{opt}}) \leq 0$ then
11: return $\tau_{\text{opt}}$, $c_2^{\text{mimo}}(\tau_{\text{opt}})$ & $k_{\text{opt}}$
12: else
13: $\tau_{\text{opt}} = \emptyset$, $c_2^{\text{mimo}}(\tau_{\text{opt}}) = \emptyset$ & $k_{\text{opt}} = \emptyset$
14: return $\tau_{\text{opt}}$, $c_2^{\text{mimo}}(\tau_{\text{opt}})$ & $k_{\text{opt}}$
15: end if
16: if $ii = \text{iter}$ then
17: return $\tau_{\text{opt}}$, $c_2^{\text{mimo}}(\tau_{\text{opt}})$ & $k_{\text{opt}}$
18: end if
19: end while

Subalgorithm-2.4 Transmit power constraint active

Require: $\tau_{\text{opt}}$, $c_1^{\text{mimo}}(\tau_{\text{opt}})$ & $k_{\text{opt}}$

1: Initialize: $i^e = 100$, $i = 1$, $\text{iter} = 100$
2: while $i < \text{iter}$ do
3: \[ i^e = i^e + 1 \]
4: Calculate: $c_3^{\text{mimo}}(\tau_{\text{opt}})$ & $c_3^{\text{mimo}}(\tau_{\text{opt}})$ Using (47) & (95)
5: if $|\tau_{\text{sum}} - \tau_{\text{opt}}| \leq 10^{-3}$ then
6: \[ i^e = \tau_{\text{opt}} \]
7: Calculate: $\eta(\tau_{\text{opt}})$, $c_3^{\text{mimo}}(\tau_{\text{opt}})$ & $c_3^{\text{mimo}}(\tau_{\text{opt}})$ Using (73), (39) and (46)
8: if $\lambda_1 \leq 0$, $c_3^{\text{mimo}}(\tau_{\text{opt}}) \leq 0$ & $c_3^{\text{mimo}}(\tau_{\text{opt}}) \leq 0$ then
9: Calculate: $k_{\text{opt}}$, $c_3^{\text{mimo}}(\tau_{\text{opt}})$ Using (47) and (70)
10: return $\tau_{\text{opt}}$, $k_{\text{opt}}$, $c_3^{\text{mimo}}(\tau_{\text{opt}})$
11: else
12: return $\tau_{\text{opt}}$, $k_{\text{opt}}$, $c_3^{\text{mimo}}(\tau_{\text{opt}})$
13: end if
14: end if
15: if $ii = \text{iter}$ then
16: return $\tau_{\text{opt}}$, $k_{\text{opt}}$, $c_3^{\text{mimo}}(\tau_{\text{opt}})$
17: end if
18: $ii = ii + 1$
19: end if
20: end while

APPENDIX B

SUBALGORITHMS FOR ALGORITHM-2

REFERENCES


