Packet Size Optimization for Cognitive Radio Sensor Networks aided Internet of Things

Chitradeep Majumdar*, Doohwan Lee†, Aaqib Ashfaq Patel*, S. N. Merchant*, U. B. Desai‡

*IIT Bombay, †RCAST, The University of Tokyo, ‡IIT Hyderabad

e-mail: cm6v07@ee.iitb.ac.in, leedh@mlab.t.u-tokyo.ac.jp, aaqib@ee.iitb.ac.in, merchant@ee.iitb.ac.in, ubdesai@iith.ac.in

Abstract—Cognitive Radio Sensor Networks (CRSN) is state of the art communication paradigm for power constrained short range data communication. It is one of the potential technology adopted for Internet of Things (IoT) and other futuristic Machine to Machine (M2M) based applications. Many of these applications are power constrained and delay sensitive. Therefore, CRSN architecture must be coupled with different adaptive and robust communication schemes to take care of the delay and energy-efficiency at the same time. Considering the tradeoff that exists in terms of energy efficiency and overhead delay for a given data packet length, it is proposed to transmit the physical layer payload with an optimal packet size (OPS) depending on the network condition. Furthermore, due to the cognitive feature of CRSN architecture overhead energy consumption due to channel sensing and channel handoff plays a critical role. Based on the above premises, in this paper we propose a heuristic exhaustive search based Algorithm-1 and a computationally efficient suboptimal low complexity Karuh-Kuhn-Tucker (KKT) condition based Algorithm-2 to determine the optimal packet size in CRSN architecture using variable rate m-QAM modulation. The proposed algorithms are implemented along with two main cognitive radio assisted channel access strategies based on Distributed Time Slotted-Cognitive Medium Access Control (DTS-CMAC) and Centralized Common Control Channel based Cognitive Medium Access Control (CC-CMAC) and their performances are compared. The simulation results reveals that proposed Algorithm-2 outperforms Algorithm-1 by a significant margin in terms of its implementation time. For the exhaustive search based Algorithm-1 the average time consumed to determine OPS for a given number of cognitive users is 1.2 seconds while for KKT based Algorithm-2 it is of the order of 5 to 10 ms. CC-CMAC with OPS is most efficient in terms of overall energy consumption but incurs more delay as compared to DTS-CMAC with OPS scheme.

Index Terms—Optimal packet size, cognitive radio sensor networks, energy-efficiency, quadrature amplitude modulation, convex optimization, medium access control.

I. INTRODUCTION

THE last few years has witnessed significant progress in the areas of wireless Sensor networks. Major technical challenges associated with the transmission of the sensed data to a remote server or gateway has been addressed by the researchers both from the perspective of the algorithms and the embedded hardware complexity. With the advent of the emerging technologies like Internet of Things (IoT) and big data which enables us to sense, transmit and process massive amount of data from our physical surroundings, the relevance of sensor networks in the context of IoT turns out to be of prime importance. Majority of these sensor nodes are power constrained operating on limited battery power or other ambient sources of power. Therefore, efficient communication and scheduling protocols must be implemented within the sensor node so that the power consumption of the sensor nodes could be minimised to enhance the network lifetime. Sensor networks could be deployed indoor or outdoor for various practical purposes like event detection and periodic monitoring of any physical phenomenon [1], smart cities [2], healthcare and body sensor networks related biomedical applications [3], [4], smart grid [5], [6] etc. An expected exponential increase in the number of WSN nodes for IoT applications in the coming few years, all sharing the same frequency band (2.4 GHz) would pose new challenges for data transmission. With the commercialization of all-in-one single system on chip (SoC) IoT solution based on 22 nm CMOS fabrication technology by the manufacturers with different communication protocols stacked into a single chip [7], the size of these sensing devices or nodes are getting smaller. Considering the scalability of these devices and the way they are likely to proliferate into our daily lives in near future, it is essential to come up with highly efficient state of the art communication protocols evolved around this cutting edge technology of IoT.

Cognitive Radio (CR) is a technology which has evolved over the last decade proposed to be used mainly for mobile communication [8]. It is based on the opportunistic use of the available frequency spectrum through dynamic spectrum access by the mobile phone users. In conventional CR, the licensed users are the ones who have been allocated a portion of the frequency band by their respective mobile operators. They are the paid subscribers for the services and coined as Primary Users (PUs). Based on the SUs transmission model, there are various paradigms described in the literature like interweave, underlay and overlay cognitive communication [9]. To address the general challenges associated with sensor networks in terms of power efficiency, delay, reliability and coexistence with similar services operating within the same unlicensed band, the concept of cognitive radio is conceived within sensor networks framework leading to the emergence of an novel paradigm of cognitive radio sensor networks (CRSN). This relatively new and novel concept is still in its evolving stages and there are number of open ended areas which still
remains unaddressed. In this paper we focus specifically on the packet size optimization for CRSN architecture. Data transmission with variable packet size depending on the network condition as compared to a fixed data packet length has its own advantages in terms of energy efficiency and delay. Under worse network conditions transmitting larger packet size could increase the retransmission energy overhead as the packet is more vulnerable to collision. Packet size too small could lead to an increase of the delay overhead due to the transmission of the overhead header, trailer and other redundant bits apart from the information bits. Therefore, there exists a clear tradeoff to in terms of energy efficiency and delay which needs to be accounted to determine the OPS and it is intuitively evident that strategy based on transmission with OPS will improve the performance of the system. Furthermore, in CRSN framework the non-cognitive users or the primary users are assumed to be different services like WiFi, Zigbee, Bluetooth, Unlicensed LTE and sensor nodes which do not have cognitive or channel sensing/switching feature. These non-cognitive users can access the available channels in the ISM band at any given time as per their application requirement and do not follow a deterministic traffic pattern. Moreover, it is very difficult to implement a centralized scheduling and access control strategy for divergent communication protocol stacks. Under this scenario it is important to focus on the cognitive radio enabled sensor nodes and adapt its various transmission parameters like modulation level, transmit power and the packet size so that an improved coexistence could be achieved. In our work we have emphasized primarily on the determination of the optimal packet size for CRSN architecture using variable rate m-QAM modulation scheme. Furthermore, we have ensured that the proposed strategy is an unified strategy which minimizes the overall energy consumption of the cognitive nodes involved during the transmission phase and simultaneously satisfy key constraints like the end to end delay, interference duration caused to the non-cognitive users, average BER and the transmit power of the cognitive sensor nodes which should not exceed typically above 20 dBm.

There are few literatures available where the authors have formulated optimization problem to determine the OPS for CRSN architecture. For computationally constrained sensor network architecture it is non-trivial to implement complex algorithms to determine OPS in real time depending upon the network conditions. Based on our extensive literature survey, no work is proposed so far which provides a low complexity robust algorithm to determine the OPS. To this end the main contributions of this paper are three folds. Firstly, a joint optimization problem is formulated based which is further simplified to determine the OPS. Secondly, two algorithms based on Exhaustive Search (E.S) assisted Algorithm-1 and low complexity Karush-Kuhn-Tucker (KKT) assisted Algorithm-2 is proposed to estimate the optimal packet size. Lastly, the proposed algorithms with its cognitive feature is incorporated into a distributed time-slotted channel access scheme to evaluate its performance. Thereafter, it is compared with a centralized CSMA/CA assisted common control channel based channel access scheme.

The remainder of this paper is organized as follows. Section II discusses about the work that has been done so far in this area available in the literature. Section III describes the system model. Section IV describes the different transmission states involved during cognitive mode of transmission and estimation of the involved channel sensing time for a given detection and false alarm threshold. Section V shows the modelling of the basic optimization problem used to determine the OPS for CRSN. Section VI describes the remodelling and simplification of the optimization problem with variable rate m-QAM based modulation scheme. Section VII describes the proposed algorithm based on exhaustive search and Newton-Raphson assisted KKT-based approach. Numerical results are described in Section VIII and Section IX concludes the paper.

II. RELATED WORKS

Authors of [10] have proposed the metric to determine OPS both for coded and uncoded system under basic sensor network architecture. In [11], basic optimization problem is proposed by the authors with variable rate m-QAM modulation scheme for WSN without any cognitive features or network constraints. Literature available where data transmission with optimal packet size instead of fixed packet length has been analyzed [3], [4]. Furthermore, there are hardly any significant work and available literatures that has addressed the issue of OPS determination when the concept of cognitive radio is introduced along with conventional sensor network architecture. Closest to our work, in [12], the authors have proposed the framework to determine the OPS for cognitive radio based sensor network architecture under fixed transmit power from the non-cognitive under fixed rate FSK modulation scheme. In our paper we have used variable rate m-QAM modulation technique. For short range communication it has already been proven in the literatures [13], [14], [15], [16] that transmission with varying modulation level consumes minimum energy. Authors of [13] have used variable rate m-QAM for point to point and MIMO based sensor network architecture to show that it is more energy efficient as compared to fixed rate data transmission. In the context of cognitive radio, we have shown in our previous work that variable rate m-QAM based CRSN framework outperforms fixed rate FSK modulation system [17]. For cognitive channel sensing, the energy based channel detection was first proposed in [18]. Authors of [19] proposed the framework to determine the probability of detection for PUs with different transmit signal and noise statistics. In [20], authors equated the probability of misdetection and false alarm under specific probability of occupancy and busy time of the PUs (non-cognitive user in this case). In [21], authors formulated the closed form solution for the detection and false alarm probabilities under low SNR regime for AWGN, rayleigh and nakagami m-fading channel. Authors in [22] provided closed form solution for these parameters for different channel conditions under noise uncertainty. Beside there are plenty of other significant literatures available which has addressed the challenges in CRSN framework with distributed spectrum sensing to enhance the performance and energy efficiency [23],[24], [25], [26]. Authors of [28], [29] have proposed and formulated optimization problem to estimate an optimal sensing
time that increases the throughput and minimizes the overall energy consumption in CRSN architecture. In [30], [31], [32], [33], the authors have addressed various issues related to the access scheme of the CRSN framework along with congestion control.

III. SYSTEM MODEL

Cognitive radio sensor networks (CRSN) framework is considered with large number of stationary sensor nodes deployed randomly either indoor or outdoor. Among these sensor nodes, $M$ number of active cognitive nodes are present within an area of an event which needs to be report its data to a remote data server. For example in Fig. 1, there are five cognitive sensor nodes in the event region. Number of available unlicensed ISM channels is considered to be $C$ each of which has bandwidth $B = 1$ MHz. The primary users in this case are the non-cognitive users assumed to be operating in same unlicensed ISM band of 2.4 GHz contending among $C$ available ISM channels for their transmission. These includes services like Zigbee, Wifi, Bluetooth, Unlicensed LTE all operating in the free ISM band. As the non-cognitive users access these channels randomly with unpredictable traffic pattern therefore, the system is highly prone to collision and packet loss. The cognitive secondary users attempts to access these set of $(C)$ available channels opportunistically when it is not occupied by the non-cognitive users. Leveraging the energy based channel detection ability from cognitive radio, these cognitive nodes could adapt its transmission parameters depending on the physical condition. Each of the $M$ cognitive nodes has $K$ bits to transmit. As shown in case A of Fig. 1, all the $M$ cognitive nodes must transmit its data within a time duration $\tau_{total}$ which must be lesser than or equal to the end end delay constraint of $\tau_{max}$. Similarly for certain regular applications, the interference duration to the non-cognitive users ($I_{nc}$) caused due to cognitive transmission could be critical. Therefore, it should be lesser than a specific threshold ($I_{max}$) as shown in case B of Fig. 1. It is assumed that all the cognitive nodes follows a distributed time slotted cognitive medium access control (DTS-CMAC) for their data transmission similar to [12].

Furthermore, depending upon the application and scenario the time duration for which the non-cognitive users experiences interference from the cognitive secondary users should be under a permissible threshold to ensure seamless quality of service for the non-cognitive users. It is shown in case B of the system architecture that $I_{nc}$ which is the ratio of the experienced interference duration to the average busy time of the non-cognitive users must be less than or equal to $I_{max}$.

The busy and idle time of the non-cognitive users are exponential random variables with Poisson distribution denoted as $L_p$ and $V_p$ with $l_p$ and $v_p$ as the mean busy and idle time. Therefore, the probability of occupancy by the non cognitive users ($Pr_{on}$) turns out to be $\frac{l_p}{l_p + v_p}$. Similarly the probability of unoccupancy $Pr_{off}$ will be $(1-Pr_{on})$ which equals to $\frac{v_p}{v_p+l_p}$. The probability of occupancy is then varied which in turn affects the average busy time of the non cognitive users. During the data transmission phase of the cognitive users, both the cognitive transmitter and the receiver experiences interference from the non cognitive users. For our analysis it is assumed that the non-cognitive users transmit complex PSK signals. Furthermore, the channel state information and the noise characteristics are known to the cognitive transceivers. In case if the channel state information is not known at the receiver, extra preamble bits could be added which marginally increases the data packet size as proposed in [16].

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IV. Cognitive radio based transmission states

Considering the fact that the cognitive users have the channel sensing capability, the transmission mode of cognitive radio empowered sensor nodes can have a number of transmission states. The two primary states involved in the CRSN transmission are the state of detection with probability ($P_d$) and the state of false alarm with probability ($P_f$). ($P_d$) implies the probability of correctly detecting the
This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2016.2619358, IEEE Access

presence of non-cognitive users in a channel. Conventionally \((P_d)\) is the measure of how effectively the primary users (PUs) operating in the licensed frequency band are protected from the unlicensed secondary users (SUs) operating in the licensed band of the PUs. In our paradigm \(P_d\) could be more appropriately described as the measure of how efficiently the secondary cognitive users are protected from the non-cognitive users both operating in the unlicensed band. \(P_f\) implies falsely detecting the presence of non-cognitive users which corresponds to the amount of missed opportunity for transmission. Mathematically these probabilities are calculated for different signal and noise statistics by the authors of [19].

Based on energy based detection, when non cognitive users are transmitting, the discrete received signal at the cognitive receiver under hypothesis \(H_1\)

\[
y(n) = s(n) + u(n) \tag{1}
\]

Hypothesis \(H_0\) is denoted as the inactive state of the non-cognitive user under which

\[
y(n) = u(n) \tag{2}
\]

Certain basic assumptions that the noise process \(u(n)\) is independent and identically distributed (iid) ZMCSGC with variance \(\mathcal{E}(|u(n)|^2) = \sigma^2_u\), the primary signal being iid with zero mean and variance \(\mathcal{E}(|s(n)|^2) = \sigma^2_s\) and \(s(n) and u(n)\) being independent processes are taken into account. Therefore for a given decision threshold \(\lambda\) and decision statistic which is the received power considered to be

\[
T(y) = \frac{1}{N} \sum_{n=1}^{N} |y(n)|^2, \tag{3}
\]

where \(N\) is the number of samples which is equal to \(f_s \tau_s\) where \(\tau_s\) is the channel sensing time and \(f_s = 2B\) from Nyquist sampling theorem.

Therefore,

\[
P_d = Pr(T(y) > \lambda|H_1) \tag{4}
\]

\[
P_f = Pr(T(y) > \lambda|H_0), \tag{5}
\]

which is equal to

\[
P_d = \int_{\lambda}^{\infty} p_0(x)dx \tag{6}
\]

\[
P_f = \int_{\lambda}^{\infty} p_1(x)dx, \tag{7}
\]

where \(p_0(x)\) and \(p_1(x)\) are the probability density functions (pdf) of the random variable test statistic \(T(y)\). The pdf of \(T(y)\) will be have \(\chi^2\) with 2\(N\) degrees of freedom as both \(s(n)\) and \(u(n)\) are complex. It is usually represented by gamma function but considering the central limit theorem for large values of \(N\), the pdfs for \(H_0\) and \(H_1\) boils down to Gaussian distribution with their corresponding means \((\mu_0, \mu_1)\) and variances \((\sigma_u^2, \sigma_s^2)\) depending upon the signal and noise statistics.

For complex PSK primary signal with complex gaussian noise, the mean and the variance for two different hypothesis \(H_0(\mu_0, \sigma_u^2)\) and \(H_1(\mu_1, \sigma_s^2)\) would be \(H_0(\sigma_u^2, \frac{1}{2}\sigma_u^2)\) and \(H_1(\gamma_{pr} + 1)\sigma_u^2, \frac{1}{2}(2\gamma_{pr} + 1)\sigma_u^2)\) [19]. Here \(\gamma_{pr} = \frac{\sigma_u^2}{\sigma_s^2}\) is the received signal to noise ratio from the non-cognitive user or \(SNR_p\). As per (6) and (7), integrating the gaussian distribution \(p_0(x)\) and \(p_1(x)\) and taking into account that the gaussian Q-function, \(Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-\frac{t^2}{2}} dt\), the \(P_d\) and \(P_f\) is estimated to be

\[
P_d = Q\left(\frac{\lambda}{\sigma_u^2} - \gamma_p - 1\right) \sqrt{\frac{\tau_s f_s}{2\gamma_p + 1}} \tag{8}
\]

\[
P_f = Q\left(\frac{\lambda}{\sigma_u^2} - 1\right) \sqrt{\tau_s f_s}, \tag{9}
\]

The effective probability of detection and false alarm will depend on the occupancy and unoccupancy of the non-cognitive users. Therefore,

\[
Pr_1 = Pr_{on}Q\left(\frac{\lambda}{\sigma_u^2} - \gamma_p - 1\right) \sqrt{\frac{\tau_s f_s}{2\gamma_p + 1}} \tag{10}
\]

\[
Pr_3 = Pr_{off}Q\left(\frac{\lambda}{\sigma_u^2} - 1\right) \sqrt{\tau_s f_s}, \tag{11}
\]

where \(Pr_1\) and \(Pr_3\) are the respective states of being into the state of detection and false alarm respectively while \(P_d\) and \(P_f\) are probabilities of detection and false alarm.

Misdetection state is the state of wrongly detecting the absence of non-cognitive user. The probability of being into the state of misdetection will be

\[
Pr_2 = Pr_{on}(1 - P_d). \tag{12}
\]

Since, \(Q(-x) = 1 - Q(x)\), therefore

\[
Pr_2 = Pr_{on}Q\left(\gamma_p + 1 - \frac{\lambda}{\sigma_u^2}\right) \sqrt{\frac{\tau_s f_s}{2\gamma_p + 1}}. \tag{13}
\]

In our system model it is assumed that the non-cognitive (PUs) are transmitting complex-PSK signal. Furthermore, it is also assumed that considering dense deployment of the non-cognitive users around he proximity of the cognitive secondary users, the probability density function of the cumulative interference power or the received SNR at the cognitive receiver from the non-cognitive user will obey gaussian distribution. Based on these assumptions, for a given probability of detection and false alarm threshold, the sensing time for a given channel SNR \((\gamma_{pr})\) is calculated to be as

\[
\tau_s = \frac{1}{2B\gamma_{pr}} \left[Q^{-1}(P_f) - Q^{-1}(P_d)\sqrt{2\gamma_{pr} + 1}\right]^2, \tag{14}
\]

where \(B\) is the channel bandwidth and \(Q(\cdot)\) is the gaussian Q-function as shown in [19] considering the sampling frequency \(f_s \geq 2B\) (Nyquist rate). In the later section of this paper \(\gamma_{pr}\) is denoted as \(SNR_{pr}\). It is the total received power from the primary user (non-cognitive user in our case) normalised by the total noise power \(N_0\) where \(B\) is the bandwidth of the sensed channel and \(N_0 = -171\) dBm/Hz is the noise power spectral density.

As mentioned before that the access scheme followed by the cognitive users is distributed time slotted medium access control, there lies a possibility that a cognitive user might
This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2016.2619358, IEEE Access

Because of the ergodic behaviour of the non-cognitive users, that more than one user does not select the same channel. The number of contending users. Pr_c represents the probability that more than one user does not select the same channel. Because of the ergodic behaviour of the non-cognitive users, there are total \( Pr_{off} C \) channels available for \( M \) users. If a single cognitive user selects a channel, other \( M-1 \) users needs to select from the remaining \( (CPr_{off} - 1) \) channels to avoid collision among multiple cognitive users whose probability is \( Pr_c \) shown in the equation above. Therefore, probability of multiple cognitive users selecting the same channel for data transmission is simply \((1-Pr_c)\). Moreover, non-cognitive user should be absent and there should be no false alarm in this state of co-selection. Hence, the probability of being into such state \( Pr_4 \) will be

\[
Pr_4 = Pr_{off} (1 - P_f) (1 - Pr_c).
\] (16)

The fifth case of Collision state is the scenario where the cognitive user correctly detects the absence of the non-cognitive user and starts transmission. Then all of a sudden the non-cognitive user appears on the same channel and continues transmission for the remaining duration of the cognitive user transmission. Therefore, the cognitive user will experience collision and interference. Probability of such an event taking place will be \( Pr(V_p \leq l_s/R) \) where \( l_s \) is the packet size in bits, \( R \) is the data rate and \( l_s/R \) denotes the transmission time by the cognitive user to transmit each data packet. It will be to \(1 - \int_{l_s/R}^{\infty} \frac{1}{\tau} e^{-\tau} dt\) considering the fact that the the pdf of random variable \( V_p \) has exponential distribution. For this scenario to hold good, the above state of co-selection should not occur. Therefore, the collision probability will be

\[
Pr_5 = Pr_{off} (1 - P_f) Pr_c e^{-l_s/\tau}. \] (17)

The last condition is the state of success where the cognitive users successfully detects the absence of the non-cognitive users and successfully delivers the data packet to the destination or to the next hop. States of co-selection and collision should not occur during this phase. The probability of success would be

\[
Pr_6 = Pr_{off} (1 - P_f) Pr_c e^{-l_s/\tau}. \] (18)

V. PROBLEM FORMULATION TO DETERMINE OPTIMAL PACKET SIZE FOR COGNITIVE ARCHITECTURE

A. Formulation and analysis of the cost function

In order to determine the optimal packet size, it is essential to define a cost function which effectively captures the aspect of overall energy consumption to transmit each data packet and the reliability of the transmission. From the perspective of cognitive radio based sensor networks architecture, apart from data transmission there are other cognitive functionalities involved like channel sensing, decision and spectrum handoff which has to be taken into account represented as \( E_{tot} \). Based on these parameters a basic cost function (packet energy efficiency reliability metric) is formulated by combining the energy efficiency of each packet transferred \( en(l_s) \) and it’s reliability \( r(l_s) \) both of which are function of packet size \( l_s \). The cost function would be the function of the packet size,

\[
CostFunction: \eta(l_s) = e_n(l_s) \times r(l_s). \] (19)

In the equation above

\[
e_n(l_s) = \frac{k_1(l_s - h)}{k_1 l_s + E_{tot}}, \] (20)

where \( k_1 \) is the energy consumption per bit and \( h \) is the packet header size.

\[
E_{tot} = E_{sens} + E_{hf} + E_{dec} + E_{add}. \] (21)

where \( E_{sens} \) is the energy consumed for channel sensing, \( E_{hf} \) is the energy consumed for channel handoff, \( E_{dec} \) is the energy consumed to reach a decision about a channel and \( E_{add} \) is the additional energy consumption involved because of the spectrum decision, handoff and other transient power consumption with the transceiver.

\[
E_{sens} = \tau_s \times P_{sens}, \] (22)

where \( \tau_s \) is the channel sensing time which is dependant on received non-cognitive SNR \( (\gamma_{pr}) \) obtained from (14) and \( P_{sens} \) is power consumption due to channel sensing nearly equal to 110 mW [30].

Energy consumed for channel switching will depend on the condition that the existing channel is sensed as busy and any one of the other \((C-1)\) available channels is unoccupied by the non-cognitive users. Therefore, it is calculated to be as

\[
Pr_{idle} = Pr_{off}(1 - P_f) + Pr_{on}(1 - P_d) \] (23)

\[
Pr_{busy} = (Pr_{off} P_f + Pr_{on} P_d)(C-1) \] (24)

\[
Pr_{sw} = (1 - Pr_{idle})(1 - Pr_{busy}). \] (25)

As per authors of [30], energy consumed for channel switching \( (E_{hf}) \) in practical applications for a relaxed scenario when the channel center frequencies are close by is around 2 mJ. Therefore average energy consumed for channel handoff for CR architecture turns out to be \( Pr_{sw} E_{hf} \).

\[
r(l_s) = 1 - PER, \] (26)

where \( PER \) is the packet error rate given by \( \left\{1 - \left(1 - \frac{1}{\tau_d}\right)^{l_s}\right\} \). \( \tau_d \) is the average BER of the cognitive transmission. Therefore, the cost function is simplified to

\[
\eta(l_s) = \frac{k_1(l_s - h)}{k_1 l_s + E_{tot}} \] (27)

It is observed that for fixed value of \( k_1 \) and \( E_{tot} \), \( e_n(l_s) \) will be a linearly increasing function with respect to \( l_s \) while for a fixed \( \tau_c \), \( r(l_s) \) will be a monotonically decreasing function.
of \(l_s\). Calculating first and second order derivative of the cost function for further analysis,

\[
\eta(l_s)' = \left\{ \frac{E_{\text{tot}}k_1 + k_1^2 h}{k_1 l_s + E_{\text{tot}}} + k_1 (l_s - h) \ln (1 - \mathcal{P}_e) \right\},
\]

\[
\eta(l_s)'' = \frac{(E_{\text{tot}} k_1 + k_1^2 h) k_1}{(k_1 l_s + E_{\text{tot}})^2},
\]

where \(Z_1(l_s)\) and \(Z_2(l_s)\) are dummy variables. Similarly,

\[
Z_1(l_s)' = \ln (1 - \mathcal{P}_e) k_1 - \frac{(E_{\text{tot}}k_1 + k_1^2 h) k_1}{(k_1 l_s + E_{\text{tot}})^2},
\]

\[
Z_2(l_s)' = \frac{(k_1 l_s + E_{\text{tot}})(1 - \mathcal{P}_e)^l_s}{(k_1 l_s + E_{\text{tot}})^2} \ln (1 - \mathcal{P}_e) - (1 - \mathcal{P}_e)^l_s k_1.
\]

Since \(\mathcal{P}_e \ll 1\) therefore, \(\ln (1 - \mathcal{P}_e) \approx 0\). Using this argument and by substituting (30) and (31) in (29), it can be easily verified that \(\eta(l_s)' < 0\) which implies that \(\eta(l_s)\) is a concave function with an unique global maxima.

Therefore, the optimal packet size turns out to be \(l_s^*\) which maximizes the following cost function as long as all the posed constraints criteria are satisfied.

\[
\text{maximize } \eta(l_s) = \frac{k_1 (l_s - h)}{k_1 l_s + E_{\text{tot}}} (1 - \mathcal{P}_e)^l_s. \tag{32}
\]

Since \(\eta\) is also a function of \(k_1\), for a fixed average BER \(\mathcal{P}_e\), packet size \((l_s)\) and \(E_{\text{tot}}\),

\[
\frac{\partial \eta}{\partial k_1} = (1 - \mathcal{P}_e)^l_s (l_s - h)(E_{\text{tot}})/(k_1 l_s + E_{\text{tot}})^2. \tag{33}
\]

Since \(l_s > h\), it is clearly observed from (33) that \(\frac{\partial \eta}{\partial k_1} > 0\) and \(\eta(k_1)\) will be an increasing function with respect to \(k_1\) when \(l_s\) is fixed.

Power consumption in any generic receiver system typically comprises of the power consumed by the power amplifier \(P_{PA}\) at the transmitter end along and different other circuit components at the transmitter and the receiver end (ADC, DAC, active filters at the transmitter and receiver side, frequency synthesizer, mixers and intermediate frequency/ low noise amplifiers for the receiver). Let \(P_c = P_{\text{rx}}x + P_{\text{tx}}x\) where \(P_c\) is the total power power consumed by the circuit components. Therefore, energy consumed per bit to transmit a given data packet is estimated to be

\[
k_1 = (P_{PA} + P_c) \frac{1}{R}. \tag{34}
\]

In case of variable rate- m-QAM modulation scheme, for a fixed symbol rate \(R_s\), the modulation level which is bits/symbol \((b)\) is varied. It has been shown by the authors of [15] extensively that transmission with variable rate m-QAM modulation strategy improves the performance of the system in terms of overall energy consumption due to data transmission specially in sensor network architecture where transmission distances involved is relatively much smaller as compared to other conventional cellular networks. Moreover, \(P_{PA} = (1 + \alpha)P_{\text{out}}\) where \(P_{\text{out}}\) transmit power of the power amplifier and \(\alpha\) is the peak to average ratio (PAR) which is dependant on the modulation level as calculated to be as \(\frac{\gamma}{\gamma - 1}\) where \(\epsilon = 3\sqrt{\frac{M-1}{M+1}}\) is dependant on the constellation size \(M_q = 2^b\) for \(b \geq 2\) and even (square constellation) [15] and \(\mu = 0.35\) is the drainage efficiency of the power amplifier. Therefore, \(P_{PA}\) is a function of \(b\) and (34) could be re-written as

\[
k_1(b) = (P_{PA}(b) + P_c) \frac{1}{b R_s}. \tag{35}
\]

It is shown in the results of [13] [15] and [16] that for a specific BER threshold \((\mathcal{P}_e)\) and short distance range, the value of \(k_1(b)\) will initially decrease up to an optimal point say \(b^*\) and then gradually increase. Although modulation level \((b)\) is discrete integer however, for the sake of argument it can be said that the \(k_1(b)\) shows a convex behaviour with respect to \(b\). Although \(P_{PA}(b)\) is monotonically increasing function of \(b\) because higher transmit power \(P_{\text{out}}\) or received SNR \((\gamma_a)\) is required to attain a fixed probability of error but if we observe (35), for a fixed packet length, increase in the modulation level decreases the transmit duration. Therefore, from \(b = 2\) till \(b = b^*\) the circuit power consumption \(P_c\) is the dominant factor which decreases the value of \(k_1(b)\) while for values \((b > b^*)\), \(P_{PA}\) becomes the dominant factor.

In terms of \(\eta\) which is function of \(k_1\) (27), it is already shown in (33) that \(\eta(k_1)\) will be a monotonically increasing function with respect to \(k_1\) which in turn is a function of the modulation level \(k_1(b)\) (35) therefore, if we intend to minimize the overall energy consumption of the system over a given span of discrete modulation levels \(b = \{2, 3, 4...9\}\) and simultaneously improve the cost function (27) to obtain the optimal packet size, rather than using the cost function (32) directly, the cost function needs to be modified to a min-max cost function.

Furthermore, considering the cognitive feature in our system model the transmit power required to attain a specific average BER threshold of \((\mathcal{P}_e)\) will be dependant on the transmit duration of the cognitive users or its packet size \((l_s)\) along with the modulation level \(b\). This is because of the fact that average BER \((\mathcal{P}_e)\) is calculated based on the instantaneous BER of different cognitive transmission states whose probabilities needs to be accounted. For example for the state of collision whose probability is \(P_{\gamma_5}\) (17), is a function of \(l_s\). Even for the state of misdetection, the error probability will be dependant on \(l_s\) (explained in the later section). Intuitively, for a given fixed \(b\), \(k_1\) will be increasing function of \(l_s\) because increase in packet size or transmit duration increases chances of packet drop and contention and the power consumption to meet the BER threshold increases. Hence, the \(P_{\text{out}}\) will be a function of \(l_s\) along with \(b\) and accordingly so will be \(k_1\) which now becomes \(k_1(l_s, b)\). Details on the estimation of the average BER and \(k_1(l_s, b)\) is illustrated in the later sections. Based on the premises above, the cost function in (32) is modified to
This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2016.2619358, IEEE Access

C. Modelling of the end to end delay constraint for the non-cognitive user

Non-cognitive users experience interference from the cognitive user mainly during the states of misdetection and collision. During state of misdetection non-cognitive user could transmit all throughout the transmission duration of the cognitive secondary user or it can vacate the channel within the transmission duration of the SUs. Probability that non-cognitive user vacate the channel within cognitive user transmission period \( Pr(I_p \leq \frac{l_s}{\tau_c}) \) is calculated to be as 

\[
(1 - e^{-\frac{P_{Roff}}{l_p}})
\]

Thus calculated must be lesser than a predefined constraint threshold \( I_{max} \). As \( I_{nc} \) is also a function of the \( P_d \) and \( P_f \) from (13), (17) and (37) therefore, the constraint function \( c_1(l_s) \) could be written as

\[
c_1(l_s, b, P_d, P_f) = I_{nc}(l_s, b, P_d, P_f) - I_{max} \leq 0
\]

C. Modelling of the end to end delay constraint for the cognitive user

It is assumed that each of the \( M \) cognitive users has \( K \) information bits to transmit. Therefore for each hop, each user will have to transmit \( \frac{K}{R_c} \) number of data packets where \( h \) is the size of the packet header. An infinite Stop and Wait ARQ scheme is considered in our case where the cognitive user continues to transmit the packet for infinite times over the data channel until it is successfully transmitted. The cognitive user waits for acknowledgement from the receiver in order to initiate transmission of the next data frame. Considering size of the acknowledge packet and its packet error rate to be negligible, the expected time to transmit one packet of data is calculated to be as

\[
E(T) = T_s \left(1 + \frac{PER}{1 - PER}\right)
\]

\[
E(T) = T_s \left(1 + \frac{1 - (1 - PER)l_s}{(1 - PER)^2}\right),
\]

where \( T_s \) = \( \frac{l_s}{\tau} \) + \( \tau_{add} \) where \( \frac{l_s}{\tau} \) is the data transmission time.

\[
\tau_{add} = \tau_d + \tau_s + \tau_{dec} + \tau_{hf} + \tau_{sl/wk}
\]

is the additional delay caused due to processing at each hop \( \tau_d \), cognitive features like channel sensing \( \tau_s \), spectrum decision \( \tau_{dec} \), channel handoff \( \tau_{hf} \) and transient time for the receiver to wake up from sleep to active mode \( \tau_{sl/wk} \). Values of \( \tau_{hf} \) and \( \tau_{dec} \) is usually very small compared to the data transmission time. \( \tau_s \) is the channel sensing time directly dependent on probability of detection \( P_d \) and false alarm \( P_f \) (14). Therefore, \( \tau_{add} \) is a function of \( \tau_s \) corresponding to \( P_d \) and \( P_f \).

Since \( \frac{P_{Roff}}{l_p} \ll 1 \), using binomial theorem,

\[
E(T) = T_s \left(1 + \frac{P_{Roff}}{l_s}\right).
\]

Since each of the \( M \) sensor nodes have total \( K \) bits to transmit through \( \pi \) number of average hops therefore, the total number of packets to be transmitted by all the \( M \) cognitive nodes and the total delay would be

\[
\tau_{total}(l_s) = \frac{MK\pi}{l_s - h} E(T)
\]

\[
\tau_{total}(l_s) = \frac{MK\pi}{l_s - h} \left(1 + \frac{P_{Roff}}{l_s}\right) \left(\frac{l_s}{\tau} + \tau_{add}\right).
\]

In our analysis it is assumed that though all the \( M \) cognitive users are accessing the set of \( C \) channels simultaneously leveraging the cognitive feature however, the gateway can process only single data packet at a time instant as shown in [12]. This is the reason for scaling the delay factor by \( M \) in the above equation (43) and (44) while calculating \( \tau_{total} \) which would be the maximum possible delay \( M \) cognitive users each of which has \( K \) bits to transmit to the gateway. However, in future work modelling of the overall delay estimation could incorporate advanced concepts like concurrent multi-packet reception at the gateway which could revoke scaling the delay factor by \( M \).

The \( \tau_{total} \leq \tau_{max} \) where \( \tau_{max} \) is the maximum permissible delay or the delay constraint threshold. The delay constraint therefore is modelled as

\[
c_2(l_s) = \tau_{total}(l_s) - \tau_{max} \leq 0
\]

Since data rate \( R = bR_s \), \( \tau_{total}(l_s) \) also being a function of \( \frac{P_{Roff}}{l_p} \) (44) and \( \tau_{add} \) is function of sensing time \( \tau_s \) which depends on \( P_d \) and \( P_f \) from (14) and (41) therefore, \( c_2 \) could be rewritten as

\[
c_2(l_s, b, P_d, P_f, \frac{P_{Roff}}{l_p}) = \frac{P_{Roff}^2}{l_s} + l_s (\frac{P_{Roff}}{l_p} \tau_{add} R + 1 - kR) + R (\tau_{add} + kh) \leq 0
\]
where \( k = \frac{\tau_{\text{max}}}{\lambda R T} \).

### D. Constraint on the transmit power

For any wireless sensor networks architecture, they are reliant either on batteries or other ambient power sources. Therefore, in practical applications there is a peak power constraint which needs to be considered. In our scenario, it is assumed that the cognitive sensor nodes could transmit with maximum transmit power of 20 dBm which is 100 mW. As explained in the earlier subsection that transmit power in our case calculated based on the rate and packet size adaptation depending on the network condition to attain a specific average BER threshold \( \overline{P}_{ee} \) which will be depending on the \( P_d \) and \( P_f \) as explained in details in the later section therefore, \( c_3 \) becomes a function of \( \overline{P}_{ee}, P_d \) and \( P_f \) along with \( b \) and \( l_s \):

\[
c_3(l_s, b, P_d, P_f, \overline{P}_{ee}) = P_{\text{out}}(l_s, b, P_d, P_f, \overline{P}_{ee}) - 0.1 \leq 0.
\]  

(47)

### E. Constraint on the average BER

In our set up instead of posing a constraint on the conventional packet error rate, we are proposing to enforce a constraint on the average BER. This is because our system architecture is based on cognitive radio where there are different BERs involved for different cognitive transmission states as explained in Section III (A) and estimation of average BER is non-trivial. Therefore, for the sake of brevity and mathematical convenience, a direct constraint on the average BER reduces the complexity of our analysis. The motivation to estimate the average BER could be established if we consider (36), (46) and (47) where this parameter is a critical component to model both the cost and constraint functions related to delay and transmit power. In our analysis, to guarantee a certain level of system performance and reliability, an upper bound constraint on the average BER threshold \( \overline{P}_{eth} \) is considered which turns out to be

\[
1 - (1 - \overline{P}_{e})^\pi \leq \overline{P}_{eth}, \quad (48)
\]

where \( \pi \) is the average number of hops and \( \overline{P}_{eth} \) is the average BER constraint for a single hop. In our simulation set up \( \pi \) is considered to be 1. Therefore the (48) turns out to be \( \overline{P}_{e} \leq \overline{P}_{eth} \). In general scenario when \( \pi > 1 \), \( \overline{P}_{e} \leq \overline{P}_{eth} \) where

\[
P_{\text{eth}} = 1 - e^{-\frac{1}{11} \log(1 - \overline{P}_{eth})}
\]

### F. The optimization problem

To guarantee protection of the non-cognitive users and to maximize the transmission opportunity by the cognitive users, the probability of detection \( P_d \geq \tilde{P}_d \) and \( P_f \leq \tilde{P}_f \) where \( \tilde{P}_d = 0.9 \) and \( \tilde{P}_f = 0.1 \) which is the benchmark as per any cognitive radio specifications. Based the on this and above subsections the optimization problem to determine the optimal packet size boils down to

\[
\begin{align*}
\min_{b} & \quad \max_{l_s} \{ \eta(l_s, b, P_d, P_f, \overline{P}_{ee}) \} \\
\text{subject to} & \quad c_1(l_s, b, P_d, P_f) \leq 0 \\
& \quad c_2(l_s, b, P_d, P_f, \overline{P}_{ee}) \leq 0 \\
& \quad c_3(l_s, b, P_d, P_f, \overline{P}_{ee}) \leq 0 \\
& \quad \overline{P}_{e} \leq 1 - e^{-\frac{1}{11} \log(1 - \overline{P}_{eth})} \\
& \quad P_d \geq \tilde{P}_d \\
& \quad P_f \leq \tilde{P}_f \\
& \quad 100 < l_s < 1000, b \in \{ 2, 3, 4,...,10 \},
\end{align*}
\]  

(49a)

where both \( b \) and \( l_s \) are discrete integers.

### VI. Determination of the average BER under variable rate M-QAM and remodelling of the optimization problem

The transmit power \( P_{\text{out}} \) is determined based on the average BER \( \overline{P}_{e} \). Therefore the energy consumption Average probability of error \( \overline{P}_{e} \) will depend on the received SNR without interference from the non-cognitive user \( (\gamma_a) \) and received signal to interference and noise ratio which is the SINR \( (\gamma_b) \) both in terms of normalized bit energy to noise ratio \( \frac{E_b}{N_0} \):

\[
\begin{align*}
\gamma_a &= |g|^2 \frac{P_{\text{rec}}}{N_0 R} \\
\gamma_b &= |g|^2 \frac{P_{\text{rec}}}{(N_0 + P_{nc}) R} \\
\end{align*}
\]  

(50)

(51)

(52)

where \( P_{\text{rec}} \) is the received power from the cognitive transmitter to the cognitive receiver, \( P_{nc} \) is the power received at the cognitive receiver from the non-cognitive user as interference and \( g \) being instantaneous channel gain component with Rayleigh distribution. Again, since \( R = b R_s \) therefore, both \( \gamma_a \) and \( \gamma_b \) will depend on the modulation level \( b \). In Section III A, it is already shown that \( \gamma_{pr} = \frac{\sigma_a^2}{\sigma_b^2} \) where \( \sigma_a^2 \) is the signal power received at the receiver end, \( \sigma_b^2 = N_0 B \) is the total noise power where \( \frac{N_0}{2} \) is single sided power spectral density and \( B \) is the bandwidth of the channel. Thus \( \sigma_b^2 = \gamma_{pr} (N_0 B) \). \( \sigma_b^2 \) is now denoted as \( P_{nc} \) in this paper.

Power received at the cognitive receiver will depend upon the transmit power along with the corresponding system and network configuration which includes the pathloss, link margin, antenna gains and system implementation losses etc. \( P_{\text{rec}} \) will be dependant on the transmit power and based on Friss law of pathloss, \( P_{\text{rec}} \) can be easily calculated to be as

\[
P_{\text{rec}} = P_{\text{out}} + G_{\text{tAB}} + G_{\text{rAB}} + K_{pl} \text{dB}
\]

(53)

where \( K_{pl} = 20 \log_{10} \left( \frac{1}{4\pi d_0^2} \right) \) is the pathloss component, \( G_{l}/G_{r} \) are the gains of the transmit and receive antennas, \( N_f \)
is the noise figure, \( M_l \) is the link margin, \( \delta \) is the pathloss exponent and \( d_0 = 1 \) is the reference distance.

Taking the absolute value of the \( P_{rec} \) and substituting in (50) and (52) we can obtain the instantaneous SNR and SINR. Taking expectation operator \( \mathbb{E} (\cdot) \) of \( \gamma_a \) and \( \gamma_b \) yields the average received SNR and SINR \( \tau_{a}^o \) and \( \eta_b^o \) which is used to estimate the average BER of the system.

As explained in the earlier section that the average BER is dependant on the probability of different cognitive transmission states, the instantaneous BER needs to be calculated for different states where cognitive nodes transmits and the instantaneous BER has to be averaged over the pdfs of the received SNR and SINR to obtain the average BER. Let \( \zeta(\gamma_a) \) and \( \zeta(\gamma_b) \) be the corresponding BERs for the SNR \( (\gamma_a) \) and SINR \( (\gamma_b) \) respectively. Therefore, the instantaneous BER for different cognitive states can be estimated as

\[
\begin{align*}
\text{Misdetetion (} \zeta_2(\gamma) & \text{):} \\
& = \left( P_{on} + P_{off} e^{\tau_{a}^o} \right) \zeta(\gamma) + \left( P_{off} \left( 1 - e^{\tau_{a}^o} \right) \right) \zeta(\gamma_a) \\
\text{Co - selection (} \zeta_4(\gamma) & \text{):} \\
& = \zeta(\gamma_b) \\
\text{Collision (} \zeta_3(\gamma) & \text{):} \\
& = P_{on} \zeta(\gamma_b) + P_{off} \zeta(\gamma_a) \\
\text{Successful transmission (} \zeta(\gamma) & \text{):} \\
& = \zeta(\gamma_a),
\end{align*}
\]

(54) - (57)

where \( \zeta(\gamma), \gamma \in \{ \gamma_a, \gamma_b \} \) is the BER expression for the variable m-QAM modulation scheme given by

\[
\zeta(\gamma) = \frac{4}{b} \left( 1 - \frac{1}{2^{b \delta}} \right) Q \left( \sqrt{\frac{3b}{M_b - 1}} \gamma \right).
\]

(58)

Instantaneous BERs obtained in (54) to (57) needs to be weighted with the probabilities of its corresponding transmission states as shown in (13), (16) to (18). Therefore, the total instantaneous BER is calculated to be

\[
\zeta_{\text{total}} = \frac{P_{2} \zeta_2 + P_{4} \zeta_4 + P_{5} \zeta_5 + P_{6} \zeta_6}{P_{on}(1 - P_d) + P_{off}(1 - P_f)}.
\]

(59)

Substituting (54) to (57) in the above equation (59) and by simplifying we obtain

\[
\zeta_{\text{total}} = \zeta(\gamma_a) + \Omega \left\{ \zeta(\gamma_b) - \zeta(\gamma_a) \right\},
\]

(60)

where

\[
\Omega = \frac{P_{2}(P_{on} + P_{off} e^{\tau_{a}^o}) + P_{4} + P_{on} P_{5}}{P_{off}(1 - P_f) + P_{on}(1 - P_d)}.
\]

(61)

The total average BER \( P_{e} \) is now calculated by integrating \( \zeta_{\text{total}} \) over the pdfs of \( \gamma_a \) and \( \gamma_b \).

\[
P_{e} = \int_{0}^{\infty} \int_{0}^{\infty} \zeta_{\text{total}}(\gamma_a, \gamma_b) \frac{1}{\gamma_a} e^{-\gamma_a} \frac{1}{\gamma_b} e^{-\gamma_b} d\gamma_a d\gamma_b.
\]

(62)

The expression for the BER in (58) could be further relaxed using Chernoff bound to obtain an upper bound on the BER which is

\[
\zeta(\gamma) \leq \frac{4}{b} \left( 1 - \frac{1}{2^{b \delta}} \right) e^{-\frac{3b}{2(M_b - 1)}} \gamma,
\]

(63)

where \( \gamma \in \{ \gamma_a, \gamma_b \} \). Since \( \int_{0}^{\infty} \frac{1}{\gamma} e^{-\gamma} = 1 \), using (60) to (63) we get

\[
P_{e} \leq \frac{4}{b} \left( 1 - \frac{1}{2^{b \delta}} \right) \left[ \left( 1 - \frac{1}{\Omega} \right) \int_{0}^{\infty} e^{-\frac{3b}{2(M_b - 1)}} \gamma_{a} d\gamma_{a} + \frac{1}{\Omega} \int_{0}^{\infty} e^{-\frac{3b}{2(M_b - 1)}} \gamma_{b} d\gamma_{b} \right].
\]

(64)

Therefore it is clear from the above equation that \( P_{e} \) will be a function of \( l_s \) since \( \Omega \) is a function of \( l_s \) from (63). It is certainly a function of \( b \) since \( R = bR_s \). Furthermore, \( \Omega \) is also a function of the probability of detection \( P_d \) and false alarm \( P_f \) as per (61). For a specific threshold of probability of detection \( P_d \) which is 0.9 in our simulation set up, the probability of false alarm \( P_f \) will be a convex monotonically decreasing function with respect to the sensing time \( T_s \) [19]. Again, for a fixed false alarm value, \( P_d \) will be an increasing function of sensing time \( T_s \). Since we have a delay sensitive networks and increasing sensing time leads to overhead energy consumption thus, the inequalities on probabilities of false alarm and detection as in (49f) and (49g) can be now considered to be equality constraints \( P_d = P_{d}^* \) and \( P_f = P_{f}^* \). Therefore, \( P_{e} \) now becomes \( P_{e}^* (l_s, b, P_{d}, P_{f}) \), a function of \( P_d \) and \( P_f \) along with \( l_s \) and \( b \). Since \( \frac{1}{\Omega} \ll 1 \), substituting \( \gamma_b \) in terms of \( \gamma_a \) from (52) we obtain

\[
P_{e}^* (l_s, b, P_{d}, P_{f}) \leq \frac{4}{b} \left( 1 - \frac{1}{2^{b \delta}} \right) \left[ \left( 1 - \frac{1}{2^{b \delta}} \right) \frac{3b}{2(M_b - 1)} \right]^{-1} \frac{1}{\gamma_a} N_0 + \Omega P_{\text{net}}.
\]

(65)

Based on the equation above (65), an upper bound on the received SNR \( (\gamma_a) \) could be obtained for a specific BER threshold \( P_{e}^* \) [34][35]. The constraint in (49e) is taken care by considering \( P_{e} = P_{e}^* \) based on which we can write SNR \( \gamma_a \) as a function of \( P_{e}^* \) which results to

\[
\gamma_a (l_s, b, P_{d^*}, P_{f^*}, P_{e}^*) \leq \frac{4}{b} \left( 1 - \frac{1}{2^{b \delta}} \right) \left[ \left( 1 - \frac{1}{2^{b \delta}} \right) \frac{3b}{2(M_b - 1)} \right]^{-1} \frac{1}{P_{e}^*} N_0 + \Omega P_{\text{net}}.
\]

(66)

Using the equation above, now we can easily calculate the received SNR at the cognitive receiver and the corresponding transmit power required to attain specific BER threshold of \( P_{e}^* \) from (53) and (66).

\[
P_{out} (l_s, b, P_{d^*}, P_{f^*}, P_{e}^*) \leq (4\pi)^2 b^3 a M_i N_f \frac{\gamma_a}{G_i G_s \lambda^2} N_0 R.
\]

(67)

Similarly, since energy consumed per bit \( (k_1) \) is depending directly on the transmit power \( P_{out} \). From (35) and (67),

\[
k_1 (l_s, b, P_{d^*}, P_{f^*}) \leq \Omega P_{\text{net}} + \frac{1}{R}.
\]

(68)

The initial cost function (36) now becomes,

\[
\eta = k_1 (l_s, b, P_{d^*}, P_{f^*}) (l_s - h) (1 - \frac{P_{e}^*}{P_{e}^*})^{\delta}.
\]

(69)

Additional energy overhead \( E_{\text{tot}} \) will be dependant on \( P_{d^*} \) and \( P_{f^*} \) because of the overhead energy consumed due to
channel sensing and channel switching during handoff. Based on $P_d$ and $P_f$, sensing time ($\tau_s$) needs to be calculated which is used in (14), (22) and (25) to estimate the respective energy consumptions during sensing and switching events. Furthermore, the constraint functions $c_1$, $c_2$ and $c_3$ will be directly dependent on $P_d$ and $P_f$ from (49b), (49c) and (49d). Both $c_2$ and $c_3$ are also dependent on $P_{eth}$. Henceforward, the optimization problem can be modified with direct constraints on average BER, probability of detection ($P_d$) and false alarm $P_f$ accounted in the cost function and the other remaining constraint. It relaxes the number of constraint functions which makes it easier to solve this non-linear NP-hard optimization problem. The final optimization problem therefore boils down to

$$\min \limits_b \quad \max \limits_{l_s} \eta(l_s, b, P_d, P_f, P_{eth})$$

subject to

$$c_1(l_s, b, P_d, P_f) \leq 0 \quad (70a)$$
$$c_2(l_s, b, P_d, P_f, P_{eth}) \leq 0 \quad (70b)$$
$$c_3(l_s, b, P_d, P_f, P_{eth}) \leq 0 \quad (70c)$$
$$100 < l_s < 1000, b \in \{2, 3, 4, \ldots, 10\} \quad (70d)$$

where both $b$ and $l_s$ are discrete integers.

VII. PROPOSED ALGORITHMS BASED ON HEURISTIC EXHAUSTIVE SEARCH (E.S) AND KKT APPROACH

A. Exhaustive Search Algorithm

In the proposed Heuristic Exhaustive Search Algorithm, the optimal value of the packet size is searched within a span of discrete packet sizes ranging from 100 to 2000 bits for varying modulation level sizes from 2 to 9. In the proposed Algorithm-1 there are two subalgorithms (Subalgorithm-1.1 and 1.2) which is used to determine the OPS value ($l^{opt}_s$) and its corresponding energy consumption per bit ($\eta^{opt}_1$) and transmit power ($P^{opt}_{out}$) for a given modulation level. In Subalgorithm 1.1 the optimal packet size $l^{opt}_s$ is obtained by maximizing the cost function taking into account the constraints related to non-cognitive interference duration ($c_1$) and delay $c_2$. If they are not satisfied the algorithm immediately selects the next higher modulation level. If both the constraints are satisfied for a given modulation level, then it checks whether the transmit power constraint $c_2$ is satisfied or not. In case if the transmit power constraint is not satisfied, it goes to Subalgorithm-1.2 to adapt the packet size further to see in case if the constraint $c_2$ could be met. In case if it manages to obtain the optimal point, it passes on the estimated value of ($l^{opt}_s$, $k^{opt}_1$) and ($P^{opt}_{out}$), then moves on to the next higher modulation level. Once it estimates these parameters for all the modulation level from 2 to 9, it selects the optimal packet size based on minimum $k^{opt}_1$ value since it is already explained in (33) and (36) that the cost function $\eta$ is an increasing function of the energy consumption per bit $k_1$ and optimization problem is a min-max problem. In this case as explained elaborately in the previous section $P_d = P_{da}$, $P_f = P_{fa}$ and $P_{eth} = P_{eth}$ for all the values of $b$ since $\pi = 1$ and energy consumption per bit $k_1$ is adapted for every values of the modulation level $b$ to attain the specific average BER $P_{eth}$. In Subalgorithm-1.1 and 1.2, \{\emptyset\} implies that the constraint functions is not satisfied and optimal packet size does not exist. In that case the cognitive node will either hand off and start sensing among any of the $(C - 1)$ available channels or back off for a random duration of time if all channels are sensed as busy. \{x=y\} implies the value $x$ corresponding to the value of some variable $y$.

B. Conventional Karush-Kuhn-Tucker (KKT) based algorithm

In this proposed algorithm we are limiting the search space by considering the packet size ($l_s$) to continuous rather than discrete integers. Since packet size cannot be continuous therefore, we find the optimal packet size $l^{*}_s$ and take the ceiling and floor of the $l^{*}_s$. Subsequently, the packet size which has minimum difference with its corresponding ceil and floor values is selected to be the optimal packet size ($l^{opt}_s$). Now in finding the optimal packet size and simultaneously meet the three other constraint functions, the Karush-Kuhn-Tucker

![Fig. 2. Heuristic exhaustive search technique based Algorithm-1](image-url)
out to be similar to that of Algorithm 1. The lagrangian function turns to be maximizes for range of KKT based strategy is adopted because the cost function is a concave function of \( l_s \) and constraint functions are non-linear. Numerically Newton-Raphson technique is used to find the root of the Lagrangian function. Based on this, the Lagrangian function is conceived with the cost function, constraint functions are non-linear.

Numerically Newton-Raphson technique is used to find the root of the Lagrangian function. Based on this, the Lagrangian function is conceived with the cost function, constraint functions are non-linear. Thereafter, the conditions to determine the KKT points which includes the condition for optimality, feasibility, complimentary slackness and negativity of the lagrangian multipliers as for a fixed modolation level \( b, \hat{P}_d, \hat{P}_f \) and \( \overline{P}_{tx} \) the cost function needs to be maximizes for range of \( l_s \) values. Rest the strategy is similar to that of Algorithm 1. The lagrangian function turns out to be

\[
\frac{\partial L(l_s, \lambda_1, \lambda_2, \lambda_3)}{\partial l_s} = \frac{\partial \eta(l_s)}{\partial l_s} + \lambda_1 \frac{\partial c_1(l_s)}{\partial l_s} + \lambda_2 \frac{\partial c_2(l_s)}{\partial l_s} + \lambda_3 \frac{\partial c_3(l_s)}{\partial l_s} = 0
\]

(71)

The KKT optimization problem (71), (72a) to (72c) can be solved using Algorithm 2 which contains four Subalgorithms (2.1, 2.2, 2.3 and 2.4). Subalgorithm 2.1 corresponds all the constraints are inactive. Algorithm 2 selects Subalgorithm-2.2 when interference duration constraint \( c_1(l_s) \) is active. Subalgorithm-2.3 when delay constraint \( c_2(l_s) \) is active and Subalgorithm-2.4 when the transmit power constraint \( c_3(l_s) \) is active. In order to estimate the lagrangian multipliers derivatives of the different constraint functions are required to be estimated. The detailed mathematical steps involved
used in various Subalgorithms to solve the above optimization problem is illustrated in the appendix section.

VIII. NUMERICAL RESULTS

Table I. SIMULATION PARAMETERS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>Channel Bandwidth</td>
<td>1 MHz</td>
</tr>
<tr>
<td>f</td>
<td>Operating Frequency</td>
<td>2.4 GHz</td>
</tr>
<tr>
<td>Pr0(ν)</td>
<td>probability of primary occupancy</td>
<td>2/3</td>
</tr>
<tr>
<td>v_p</td>
<td>avg. channel idle time</td>
<td>200 ms</td>
</tr>
<tr>
<td>d_s</td>
<td>distance between two cognitive secondary users</td>
<td>8 m</td>
</tr>
<tr>
<td>b</td>
<td>bits/symbol for m-QAM</td>
<td>[2,3,9]</td>
</tr>
<tr>
<td>R_s</td>
<td>fixed symbol rate</td>
<td>10 bauds</td>
</tr>
<tr>
<td>δ</td>
<td>path-loss exponent</td>
<td>2.25</td>
</tr>
<tr>
<td>G_t G_r</td>
<td>Gain of the transmitter and receiver of secondary users</td>
<td>5 dB</td>
</tr>
<tr>
<td>M_1</td>
<td>link margin and noise figure</td>
<td>5 dB and 10 DB</td>
</tr>
<tr>
<td>P_t cx</td>
<td>Power consumed by the transmitter circuit</td>
<td>98.3 mw</td>
</tr>
<tr>
<td>P_c cx</td>
<td>Power consumed by the receiver circuit</td>
<td>125 mw</td>
</tr>
<tr>
<td>N_0</td>
<td>one sided thermal noise</td>
<td>-171 dBm/Hz</td>
</tr>
<tr>
<td>header(h)</td>
<td>size of the header in bytes</td>
<td>6 bytes</td>
</tr>
<tr>
<td>P sens</td>
<td>power consumed by the circuit due to channel sensing</td>
<td>100 mw</td>
</tr>
<tr>
<td>E_n f</td>
<td>energy consumed during hand-off</td>
<td>2 mJ</td>
</tr>
</tbody>
</table>

In (45) it is shown that the average end to end delay should be lesser than or equal to the delay constraint $\tau_{\text{max}}$. In our simulation set up we are considering $\tau_{\text{max}} = M\bar{n}_f K \frac{K}{N_f}$ where $K=10$ Kbits is the total number of bits required to be transmitted by each cognitive sensor node, $R_s$ is the baud rate at 10 Ksymbols/s and $\bar{n}$ is the average number of hops which is assumed to be 1 for the sake of simplicity. As the $s_f$ decreases, the delay constraint ($\tau_{\text{max}}$) decreases which implies that the delay constraint is becoming more severe. It could be observed that OPS increases as $s_f$ decreases beyond a certain number of users. For $s_f = 0.8$ and 0.7 it could be seen that OPS value is almost same till $M=4$ but when number of users is equal to 5, the OPS value changes at $s_f = 0.7$ is 404 and $s_f = 0.8$ is 398 bits. This gap would increase with increasing $M$ as OPS at $s_f = 0.7$ will saturate at 404 bits while it will keep on decreasing at $s_f = 0.8$. This is because of the fact that at $s_f = 0.8$, the delay constraint is actually becoming inactive. Therefore with increasing number of users in the system, based on similar arguments provided for Fig. 4(a) the OPS value will keep on decreasing. However, when $s_f = 0.7$, the delay constraint becomes active beyond 4 users. In order to satisfy the delay constraint based on proposed Algorithm 1 and Algorithm 2, it would try to search for the packet size $l_s$ for a given modulation level which could satisfy the delay constraint at higher packet size before deciding that OPS does not exist for that given modulation level thus switching over to the next higher modulation level $(b)$. Although in this case the cost function will not be the maximum but it is extremely important to check throughout the range of $l_s$ for a given modulation level as higher modulation level may lead to higher energy consumption.

In Fig. 4(d), the OPS value is shown for varying percentage of interference duration time for the non-cognitive uses when $SNR_{pr} = -15$ dB and channels available C=30, delay constraint inactive. The OPS value reduces as the interference to non-cognitive user duration constraint $I_{\text{max}}$ reduces and becomes more severe from 6% to 4%. From (37), the total interference duration to non-cognitive user $\tau_{\text{inf}}$ is an increasing function of $l_s$. Furthermore, larger packet size leads to greater transmit duration that increases the probability of collision $P_{r5}$ from (17). Therefore, the OPS decreases sharply for example 200 bits at $M=5$ for $I_{\text{max}}=4\%$.

In Fig. 4(e), OPS is determined for varying hop distance for different pathloss exponents at $SNR_{pr} = -10$ dB and -15 dB. OPS decreases with increasing distance in meters and pathloss exponent. This is rather straight forward to analyze as increasing or pathloss exponent increases the over pathloss thus reducing the overall received average signal to noise ratio $(\delta)$ (53). Thus more transmit power or energy will be required to attain the BER threshold thus OPS decreases. Fig. 4(f) is rather the most significant and important result of this overall figure. The MATLAB based command CPUTIME is used to estimate the elapsed CPU time to execute Algorithm-1 and Algorithm-2 separately for varying number of users. The simulation is carried out for 5000 rounds and the average is estimated. It could be seen that Exhaustive Search (E.S) based Algorithm-1 takes 1.2 seconds on average while Algorithm-2 is of the order of 5 to 10 ms. It is a significant improvement
in favour of Algorithm 2 in terms of its complexity.

The OPS value is compared for a fixed number of users in the system \( M = 6 \) with \( C = 30 \) available channels at \( SNR_{pr} = -15 \) dB in Fig. 5(a) and -20 dB in Fig. 5(b) for increasing average busy time of the non-cognitive users \( l_p \) from 100 to 400 ms, both delay and interference duration constraint relaxed. The results are shown for three different average idle time \( v_p \) at 100, 200 and 300 ms. In Fig. 5(a), with increasing \( l_p \), it is observed that the optimal packet size increases, for a given value of \( v_p \). This is due to the fact that with increasing \( l_p \), the probability of occupancy (\( Pr_{on} \)) of the non-cognitive users increases. Increased \( Pr_{on} \) increases chances of misdetection therefore, the optimal packet size should intuitively decrease as more amount of energy per bit is consumed to attain a specific BER at higher \( Pr_{on} \) value. But in this case the OPS value is increasing at \( SNR_{pr} = -15 \) dB which is counter intuitive and not obvious. This is due to the fact that with increasing \( Pr_{on} \), energy consumed due to spectrum handoff increases as \( P_{Pr_{on}} \) increases (25). Therefore, the energy due to channel switching becomes a dominant factor which reduces the value of the cost function as \( \eta \) is dependent on \( E_{tot} \) which contains \( E_{hf} \) (21) and optimal \( l_p \) is achieved at higher values. Similarly, as \( v_p \) is increased from 100 ms to 300 ms, it could be seen that the OPS value reduces. When \( l_p \) is 200 ms, the OPS value at \( v_p = 100 \) ms is 411 bits while 399 bits at \( v_p = 300 \) ms. It is based on similar argument as explained that with increasing \( v_p \), the probability of occupancy decreases for a fixed \( l_p \) since \( Pr_{on} = 1 - \frac{v_p}{l_p} \). The \( E_{hf} \) thus increasing the cost function. Therefore, the OPS value decreases. In Fig. 5(b), as the \( SNR_{pr} \) is further reduced to -20 dB, it could be seen that the trend has completely changed as compared to Fig. 5(a) because as \( SNR_{pr} \) reduces, the the channel sensing energy \( E_{sens} \) becomes more dominating factor as sensing time (\( \tau_s \)) increases with lower channel SNR for energy based channel sensing in cognitive radios. Furthermore, \( \tau_s \) is not dependant on \( Pr_{on} \) from (14). Therefore, as \( SNR_{pr} \) reduces optimal packet size will reduce with increasing average busy time and increases with increasing average idle time \( v_p \). In Fig. 5(c), the OPS value is compared for varying channel sensing time for fixed values of \( P_d \) set at 0.95, 0.9 and 0.8 for \( SNR_{pr} = -15 \) dB and \( SNR_{pr} = -10 \) dB. As sensing time increases, the energy consumption for channel sensing increases and the probability of false alarm decreases. The channel sensing energy \( E_{sens} \) is a dominating factor which reduces the cost function in this case thus increasing the optimal packet size (OPS). Furthermore, at \( SNR_{pr} = -15 \) dB it could be seen that sensing time becomes dominant beyond 3 users. As \( P_d \) increases, OPS value becomes greater as increasing \( P_d \) improves reliability of the system and reduces chances of collision with the non-cognitive users. Therefore, the \( k_1 \) reduces to attain the BER threshold resulting in lower
OPS.

For the cognitive architecture, all our analysis so far was based on Distributed time slotted cognitive medium access control (DTS-CMAC). Finally in Fig. 5(d), Fig. 5(e) and Fig. 5(f), our model is extended to a CSMA/CA assisted common control channel based cognitive MAC scheme (CC-CMAC) [31] and their performance is analyzed along with a non-cognitive system using a distributed MAC scheme without any kind of cognition and OPS mode of transmission. In CC-CMAC model, there is a dedicated control channel for the cognitive users which is never used by the non-cognitive users and the remaining data channels are used both by the cognitive and non-cognitive users. In common control channel (CC), the cognitive transmitter and the receiver negotiates the RTS and CTS packets to establish the link over a particular data channel. Since the common control channel could be busy as it is being occupied by one cognitive (Tx-Rx) pair, the remaining \((M - 1)\) users usually backs-off for a given duration of time and mean overhead delay over the control channel \(\bar{T}_{bo}\) is based on number of users, maximum number of retransmissions, maximum contention window size, slot duration, DIFS and SIFS slot lengths and time required to transmit the RTS and CTS packets by Tx and Rx as shown in (5) of [31]. Hence, in this model the probability of co-selection \(P_{R1}\) can the relaxed and rest mathematical analysis remains the same as discussed in the earlier sections for DTS-CMAC. \(\bar{T}_{bo}\) must be added to \(\tau_{add}\) (41). In Fig. 5(d) OPS value is determined for DTS-CMAC and CC-CMAC channel access schemes. It could be seen that for CC-CMAC, the OPS value does not vary with change in the number of users in the system. This is because with \(P_{R1}\) being relaxed which was function of \(M\) and key component of the average BER in DTS-CMAC access scheme, the energy consumed per bit \(k_{1}\) to satisfy specific BER threshold will no longer depend on \(M\). In Fig. 5(e) overall energy consumption is determined for these two access schemes for varying hop distances from 4 to 14 m. The overall energy consumption is analyzed where the results are quite obvious that CC-CMAC with optimal packet size transmission minimizes the overall energy consumption as compared to OPS with DTS-CMAC as CC-MAC does not depends on \(M\) or \(C\) in terms of its energy consumption. Moreover, the DTS-CMAC and CC-CMAC transmission strategies with fixed packet size without OPS is shown as well. Fixed packet length of smaller packet size of 200 bits and larger packet size of 800 bits are considered. For both the channel access schemes, it could clearly seen in Fig. 5(e)
that transmission strategy with optimal packet size obtained either from Algorithm-1 or Algorithm-2 will lead to minimum overall energy consumption as compared to transmission with fixed packet size which is significant for any sensor networks architecture. Furthermore, the naive scheme without any kind of cognitive feature would lead to larger energy consumption as compared to the cognitive scheme. This is due to the fact that for non-cognitive scenario, the transmitting nodes are not aware of any of the statistical parameter like probability of occupancy or average busy time of neighbouring services and it would transmit at a fixed transmit power which is assumed to be 5 dBm with QPSK modulation scheme ($\gamma_{pr}$) in our numerical results. Therefore, power or rate adaptation is not possible resulting into increased energy consumption. Similar trend is observed even if we increase the fixed transmit power for non-cognitive mode of transmission by the participating sensor nodes.

In Fig. 5(f), the overall delay analysis is shown for DTS-CMAC, CC-CMAC and system without cognition & OPS. Because of the mean overhead delay due to negotiation over the common control channel, the delay in case of OPS based CC-CMAC will be more than OPS based DTS-CMAC. Both these schemes beyond 8 in distance will outperform the non-cognitive/OPS based system by a significant margin. Similar trend is observed for DTS-CMAC and CC-CMAC with fixed packet size. In terms of delay Therefore, if the system is highly delay sensitive then it is better to opt for DTS-CMAC with OPS. Else if the delay constraint is inactive and overall energy consumption is the crucial factor, CC-CMAC with OPS is a better option.

IX. CONCLUSION AND FUTURE WORKS

This paper proposed a novel optimal packet size determination framework for cognitive radio based sensor networks. An optimization model is framed to determine the OPS which apart from determining the OPS, guarantees the minimum energy consumption. Two key algorithms are proposed to evaluate the OPS. From the simulation results it could be seen that the elapsed CPU time for the KKT based Algorithm-2 outperforms Algorithm-1 by a significant margin. The CPU elapsed time for Algorithm-2 is of the order of 5 to 10 ms while for Algorithm-1 it is 1.2 seconds. Although this analysis is shown in MATLAB but it is highly imperative that when these algorithms are implemented in hardware, Algorithm-2 would be a feasible option. Our algorithms is introduced to a centralized common control channel based strategy to compare its performance with the distributed one. Through extensive numerical simulations it is established that the distributed time slotted cognitive channel access scheme (DTS-CMAC) with optimal packet size is the best transmission strategy when the application is highly delay sensitive as it incurs minimum delay. In normal scenario, the CSMA/CA assisted centralized common control channel based cognitive access scheme (CC-CMAC) with OPS will outperform the distributed (DTS-CMAC) with or without OPS in terms of overall energy consumption. Both the cognitive access schemes with OPS will outperform the access strategy without cognition in terms of overall energy consumption and end to end delay. However, in future more efficient access strategies could be coupled with our proposed OPS scheme. Different deployment strategy based on the stochastic geometry can be considered for more precise analysis. It could be observed that as the channel SNR ($\gamma_{pr}$) reduces, that results in increase in sensing time which is unsuitable for delay sensitive applications. Therefore, in our future work we propose to introduce our concept to variable sensing time.

APPENDIX A

FOR SUBALGORITHM-2.1

$$\frac{\partial \eta}{\partial l_s} = \frac{(1 - \overline{Pr_{\text{eth}}})}{k_1 l_s + E_{\text{tot}}} \left( \frac{Z_3(l_s)}{(k_1 l_s + E_{\text{tot}})} + k_1 (l_s - h) \log(1 - \overline{Pr_{\text{eth}}}) \right)$$

(73)

where $Z_3(l_s)$ is the dummy variable given by

$$Z_3(l_s) = E_{\text{tot}} l_s k_1^4 + E_{\text{tot}} k_1 h E_{\text{tot}} k_1^4 + h k_1^4,$$

(74)

where

$$k'_1(l_s) = (1 + \alpha) P'_{on}(l_s).$$

(75)

From (67), it is equivalent to

$$k'_1(l_s) = Z_3 l_s.$$

(76)

Similarly,

$$k''_1(l_s) = Z_3 l_s.'$$

(77)

where

$$Z_4 = (1 + \alpha)^2 \left( 1 - \frac{1}{2^2} \right) \left( \frac{3b}{2(M_0 n_0 - 1)} \right)^{(-1)} \left( \frac{4a^2 b_0^2 M_0 n_0}{(a_0 b_0 M_0 n_0)} \right) \frac{1}{P_{on}} P_{on}.$$

(78)

$$\Omega'(l_s) = \frac{1 - \overline{Pr}}{P_{off}(1 - \overline{Pr})} + \frac{1 - \overline{Pr}}{P_{on}(1 - \overline{Pr})}.$$

(79)

$$\Omega''(l_s) = \frac{1 - \overline{Pr}}{P_{off}(1 - \overline{Pr})} - \frac{1 - \overline{Pr}}{P_{on}(1 - \overline{Pr})}.$$

(80)

Therefore the cost function is further simplified to be used for the implementation of Newton-Raphson based OPS determination in Algorithm-2 (Subalgorithm-2.1).

$$f(l_s) = Z_1 + k_1 (l_s - h) \log(1 - \overline{Pr_{\text{eth}}})(k_1 l_s + E_{\text{tot}})$$

(81)

Similarly, the derivative of $f(l_s)$ could be expressed as

$$f'(l_s) = Z_1 + \log(1 - \overline{Pr_{\text{eth}}})(k_1 l_s + E_{\text{tot}})(l_s k_1^4 + k_1(l_s - h k_1^4) + k_1(l_s - h) \log(1 - \overline{Pr_{\text{eth}}})(k_1^4 + k_1),$$

(82)

where

$$Z_3(l_s) = E_{\text{tot}} l_s k_1^4 + k_1 l_s^3 - h k_1^4 + 2k_1^4.$$

(83)

APPENDIX B

SUBALGORITHMS FOR ALGORITHM-1 AND ALGORITHM-2
Subalgorithm-1.1 Exhaustive Search with interference duration and delay constraints.

Require: \( l_{opt}, k_{opt} \) & \( P_{out}^{opt} \)
1: Initialize: \( l_1 = 100 \) to 2000, \( P_d = P_d = 0.9 \), \( P_f = P_f = 0.1 \)
2: for \( j = 1 \) to length\( (l_{opt}) \)
3: \( C_{calc} = (\lambda(l_1(j)), c_1(l_1(j))), c_2(l_1(j)), P_{out}(l_1(j)), k_1(l_1(j)) \)
4: if \( l_1(j) \leq 0 \) then
5: \( l_{opt} = 0 \)
6: if \( l_1(j) \leq 0 \) then
7: \( P_{out}^{opt} = 0 \)
8: if \( l_1(j) \leq 0 \) then
9: \( k_{opt} = 0 \)
10: end if
11: \( \max (\lambda(l_1(j))) = l_{opt} \)
12: \( P_{out}^{opt} = P_{out}(l_{opt}) \)
13: \( k_{opt} = k_1(l_{opt}) \)
14: end if
15: end if

Subalgorithm-1.2 Exhaustive Search with transmit power constraint.

Require: \( l_{opt}, k_{opt} \) & \( P_{out}^{opt} \)
1: if \( P_{out}^{opt} > P_{out} \) then
2: \( l_{opt} = \emptyset \)
3: \( k_{opt} = k_1 \)
4: return \( l_{opt}, k_{opt} \)
5: \( P_{out}^{opt} = P_{out}(l_{opt}) \)
6: end if

Subalgorithm-2.1 Both delay and interference duration constraints inactive.

Require: \( l_{opt}, c_1(l_{opt}) \), \( c_2(l_{opt}) \), \( c_3(l_{opt}) \) & \( k_{opt} \)
1: Initialize: \( l_1 = 100 \), \( i = 1, iter = 100 \), \( P_d = P_d = 0.9 \), \( P_f = P_f = 0.1 \)
2: while \( i \leq iter \) do
3: \( l_{opt} = l_1 \)
4: if \( l_1 \leq 10^{14} \) then
5: \( P_{out} = argv\min \left\{ l_1(k_1(l_1)) \right\} 
6: c_1(l_{opt}), c_2(l_{opt}) \), \( c_3(l_{opt}) \) & \( k_{opt} \)
7: Using (39), (46), (66) & (67)
8: \( P_{out}^{opt} = P_{out}(l_{opt}) \)
9: \( k_{opt} = k_1(l_{opt}) \)
10: end if
11: \( i = i + 1 \)
12: end if
13: end while

Subalgorithm-2.2 Interference duration constraint active and delay constraint inactive.

Require: \( l_{opt}, c_1(l_{opt}) \) & \( k_{opt} \)
1: Initialize: \( l_1 = 100 \), \( iter = 100 \), \( P_d = P_d = 0.9 \), \( P_f = P_f = 0.1 \)
2: while \( i \leq iter \) do
3: \( l_{opt} = l_1 \)
4: \( P_{out} = argv\min \left\{ l_1(k_1(l_1)) \right\} 
5: c_1(l_{opt}), c_2(l_{opt}), c_3(l_{opt}) \) & \( k_{opt} \)
6: Using (39), (46), (66) & (67)
7: end if
8: \( i = i + 1 \)
9: end if
10: end while

Subalgorithm-2.3 Interference duration constraint inactive and delay constraint active.

Require: \( l_{opt}, c_1(l_{opt}) \) & \( k_{opt} \)
1: Initialize: \( P_d = P_d = 0.9 \), \( P_f = P_f = 0.1 \)
2: if \( \lambda_{2} \leq 0 \) then
3: \( P_{out} = P_{out}(l_{opt}) \)
4: \( k_{opt} = k_1(l_{opt}) \)
5: end if
6: \( \lambda_{2} = \frac{P_{out}}{\lambda_{2}} \)
7: \( \lambda_{1} = \frac{P_{out}}{\lambda_{1}} \)
8: end if
9: \( \lambda_1 = \frac{P_{out}}{\lambda_1} \)
10: end while
Subalgorithm 2.4: Transmit power constraint active

Require: $t_{opt}^{r}$, $c_1(t_{opt}^{r})$ & $k_1^{opt}$

1. Initialize: $t_1^* = 100$, $i = 1$, $i_{ter} = 100$
2. while $i < i_{ter}$ do
3. $t_1^* = t_1^* + l_i B_i$
4. Calculate: $c_2(t_{opt}^{r})$ & $c_3(t_{opt}^{r})$
5. Using (47), (66), (67) & (92)
6. if $[s_{n+1} - s_n] \leq 10^{-4}$ then
7. Calculate: $\eta(t_{opt}^{r})$, $c_1(t_{opt}^{r})$ & $c_2(t_{opt}^{r})$
8. Using (73), (39) and (46) \[ \lambda_3 = \frac{c_3(t_{opt}^{r})}{c_2(t_{opt}^{r})} \]
9. if $\lambda_3 \leq 0$, $c_1(t_{opt}^{r}) \leq 0$ & $c_2(t_{opt}^{r}) \leq 0$ then
10. Calculate: $k_1^{opt}$ & $c_3(t_{opt}^{r})$
11. using (47), (67), (68) & (69)
12. return $t_{opt}^{r}$, $k_1^{opt}$ & $c_3(t_{opt}^{r})$
13. else
14. if $ii \equiv i_{ter}$ then
15. $t_{opt}^{r} = \emptyset$, $k_1^{opt} = \emptyset$ & $c_3(t_{opt}^{r}) = \emptyset$
16. return $t_{opt}^{r}$, $k_1^{opt}$ & $c_3(t_{opt}^{r})$
17. else
18. $ii = ii + 1$
19. end if
20. end if
21. end while

Appendix C

For Subalgorithms 2.2, 2.3 and 2.4

\[
\begin{align*}
\ell'_1(i) &= A_1 + iB_1, \\
\text{where} & A_1 = P_{on} + \left( P_{r2}P_{off} e^{\frac{\tau}{\tau_{th}}} \right), B_1 = \frac{1}{(P_r2 + P_{r3})^2} \left\{ \left( P_{r2} + P_{r3} \right) P_{r2}P_{off} e^{\frac{\tau}{\tau_{th}}} \frac{1}{R_{th}^{\pi}} + P_{r3}P_{off} e^{\frac{\tau}{\tau_{th}}} P_{r2}^{\pi} \right\} \\
\text{and} & P_{r3}' = \frac{1}{R_{th}^p} (P_{r3} - P_{r2}) e^{\frac{\tau}{\tau_{th}}} \\
\text{Using (46),} & A = \frac{P_{on}}{(\tau_{eh}) + kh} \text{,} B = \frac{P_{on}(\tau_{add}) R + 1 - kR + C_u}{\tau_{eh}(\tau_{add}) + kh} \text{and} C_u = \\
\ell'_2(i) &= 2l_i \frac{P_{on}}{(\tau_{eh}) + kh} + \frac{P_{on}(\tau_{add}) R + 1 - kR}{\tau_{eh}(\tau_{add}) + kh} \\
\text{Using (66), (67), (79) and (80)} \begin{align*}
\ell'_3(i) &= 4 \left( 1 - \frac{1}{2^\pi} \right) \left( \frac{3b}{\pi(M+1)} \right)^{-1} \frac{\gamma \nu_2^2 d_1^2 N_f}{(G_{th}^2 eff)} P_{n_i}(\ell'_3(i)) \\
\ell''_3(i) &= 4 \left( 1 - \frac{1}{2^\pi} \right) \left( \frac{3b}{\pi(M+1)} \right)^{-1} \frac{\gamma \nu_2^2 d_1^2 N_f}{(G_{th}^2 eff)} P_{n_i}(\ell''_3(i))
\end{align*}
\end{align*}
\]

References


