Optimal Resource Allocation for CoMP based Cellular Systems with Base Station Switching

Yoghitha Ramamoorthi and Abhinav Kumar,
Department of Electrical Engineering, Indian Institute of Technology Hyderabad, Telangana, 502285 India.
Email: {ee15resch02007, abhinavkumar}@iith.ac.in

Abstract—Base station switching (BSS) can result in significant reduction in energy consumption of cellular networks during low traffic conditions. We show that the coverage loss due to BSS can be compensated via coordinated multi-point (CoMP) based transmission in a cluster of base stations. For a BSS with CoMP based system, we propose various BSS patterns to achieve suitable trade-off between energy efficiency and throughput. We formulate the CoMP resource allocation and $\alpha$–Fair user scheduling as a joint optimization problem. We derive the optimal time fraction and user scheduling for this problem. We utilize these results to formulate the BSS with CoMP as an optimization problem. A heuristic that solves this problem for a given rate threshold is presented. Through extensive simulations, we show that suitable trade-offs among energy, coverage, and rate can be achieved by appropriately selecting the BSS pattern, CoMP cluster, and rate threshold.

Index Terms—$\alpha$–Fair throughput, base station switching (BSS), cellular network, coordinated multi-point (CoMP) transmission, downlink, energy.

I. INTRODUCTION

The significant increase in demand of data has led to deployment of a huge number of base stations (BSs) in cellular networks. The BSs consume nearly 80% of the total energy consumed in cellular networks [1], out of which 70% is consumed by power amplifiers, processing circuits, and air conditioners [2]. These BSs are typically designed and deployed for peak user demands. However, it has been shown in [3] that the user demand varies with time resulting in underutilized BSs and switching BSs during low user demand results in significant energy savings. Further, in [4], it has been shown that around 2% of global Carbon emission is from cellular networks. Thus, base station switching (BSS) during low user demand is advantageous from both economical and ecological reasons, i.e., reduction in energy consumption and Carbon footprint of the network, respectively.

In [3], a dynamic BSS strategy has been studied based on the spatial and temporal traces of real-time downlink traffic. It has been shown in [5] that up to 30% energy can be saved in a cellular network through BSS. In [6], the energy and throughput trade-offs for a given coverage have been evaluated. To overcome the coverage constraint in BSS, infrastructure sharing through multi-operator service level agreements has been proposed in [7]. A small cell based approach for BSS has been presented in [8] and [9].

A promising approach for increasing edge users performance (equivalently coverage) in cellular networks is coordinated multi-point (CoMP) based transmission and reception.

A coverage probability based analysis of CoMP systems using stochastic geometry has been derived in [10]. Further, in [11], it has been shown through analysis that CoMP can improve coverage up to 17%. The resource allocation for CoMP has been presented in [12]. A new scheduling policy for two tier CoMP network with one macro-cell and multiple small cells is proposed in [13]. However, BSS with CoMP has recently been studied.

A stochastic geometry based analysis of outage and coverage probabilities for BSS with CoMP has been performed in [14]. In [15], the outage probability for a hexagonal grid model of BSS with CoMP in terms of signal-to-noise-ratio (SNR) has been derived. The energy efficiency analysis of CoMP with BSS, under the constraint that only one BS can be switched off, has been obtained in [16]. The fundamental trade-off between energy efficiency and spectral efficiency for BSS with CoMP taking backhaul power consumption into account has been discussed in [17]. The performance of BSS with CoMP taking only uplink into consideration has been recently investigated in [18]. However, joint resource allocation for CoMP and user scheduling along with BSS has not been studied. This is the motivation of this work.

The contributions of this paper are as follows.

- Given an $\alpha$–Fair scheduler, optimal user scheduling is derived for CoMP and non-CoMP users.
- The optimal resource allocation for a CoMP cluster is derived.
- Various CoMP configurations and BSS patterns are proposed and compared.
- The joint resource allocation and user scheduling for BSS with CoMP is formulated as an optimization problem.
- A dynamic heuristic is proposed that solves the optimization problem for an energy efficient point of operation without compromising on coverage or user rates.

The organization of the paper is as follows. The system model is described in Section II. The BSS problem is formulated in Section III. In Section IV, CoMP resource allocation and user scheduling problem is presented as an optimization problem along with the derivation of the optimal solution. The BSS with CoMP optimization problem is framed in Section V. A novel heuristic that solves the BSS with CoMP problem is described in Section VI. Extensive numerical results are presented in Section VII. Some concluding remarks along with possible future works are discussed in Section VIII.
II. SYSTEM MODEL

A. Benchmark System

We consider a homogeneous OFDMA based LTE cellular network as shown in Fig. 1. The set of BSs and corresponding sectors in the network are denoted by \( \mathcal{B} = \{1, 2, \ldots, B\} \) and \( S = \{1, 2, \ldots, S\} \), respectively. Note that the BSs are represented by triangles in Fig. 1. The hexagons represent the corresponding sectors of a BS such that each BS has three sectors. Without any loss of generality, we assume that the set of sectors is ordered with the set of BSs. Hence, any BS \( b \in \mathcal{B} \) corresponds to the sectors \( 3b-2, 3b-1, \) and \( 3b \), in the set \( S \). For example, in Fig. 2, BS 4 corresponds to sectors 10, 11, and 12. We denote the set of users in the system by \( \mathcal{U} = \{1, 2, \ldots, U\} \). We consider that the users are uniformly distributed in the system for a given user density \( \mu \). Let \( M = \{1, 2, \ldots, M\} \) denote the set of subchannels available in the network. We consider a reuse factor of 1. Hence, a total of \( M \) subchannels are allotted to each sector in \( S \). A comprehensive list of mathematical notations used in this paper is presented in Table I. Next, we present the channel model considered in this paper.

B. Channel Model

We consider a time division duplex (TDD) system. For mathematical brevity, we assume a frequency flat channel model and focus on the downlink. However, a similar analysis is possible for a frequency selective channel and uplink. The downlink signal-to-interference-plus-noise ratio (SINR) of a user \( u \) from a sector \( s \), denoted by \( \gamma_{u,s}^m \), on a subchannel \( m \) is given as

\[
\gamma_{u,s}^m = \frac{P_{u,s}^m h_{u,s}^m}{\sum_{i \neq s} P_{i,s}^m h_{i,s}^m + \sigma^2},
\]

where, \( P_{u,s}^m \) is the power allocated to the subchannel \( m \) by the sector \( s \), \( \sum_{i \neq s} h_{i,s}^m \) is the interference on the subchannel \( m \), \( \sigma^2 \) is the noise power, and \( h_{u,s}^m \) denotes the channel gain between the sector \( s \) and the user \( u \). The channel gain is given by

\[
h_{u,s}^m = 10 \log_{10} \left( \frac{-PL(d)+G_s(\phi)+G_u-\nu-\rho}{10} \right),
\]

where, \( G_s \) is the antenna gain, \( \nu \) is the penetration loss, \( \rho \) is the slow fading, \( PL(d) \) is the path loss for the distance \( d \) between \( u \) and \( s \), and \( G_s(\phi) \) is the directivity gain equal to

\[
G_s(\phi) = 25 - \min \left\{ 12 \left( \frac{\phi}{\pi/2} \right)^2, 20 \right\}, \quad \forall \pi - \pi \leq \phi \leq \pi,
\]

in which \( \phi \) denotes the angle between the \( u \) and the main lobe orientation of \( s \).

C. Resource Allocation and User Scheduling

Let \( P_{BS} \) denote the total transmit power of a BS. Then, given that the BS transmit power is shared among the three sectors of a BS, the power allocated in a sector \( s \) per subchannel \( m \), \( P_{s,m} \), is given by

\[
P_{s,m} = \frac{P_{BS}}{3M}, \quad \forall \ s \in S, \ m \in M.
\]

We use \( \eta(\gamma_{u,s}^m) \) to denote the spectral efficiency achieved by a user in bits/symbol/Hz. The value of \( \eta(\gamma_{u,s}^m) \) obtained from an adaptive modulation and coding scheme (MCS) is given in Table II for various ranges of SINR [20]. Given \( \gamma_{u,s}^m \) as in (1), the link rate for the user \( u \) from sector \( s \), denoted by \( r_{u,s} \), is expressed as

\[
r_{u,s} = \eta(\gamma_{u,s}^m) \frac{SC_{OFDM} SY_{OFDM}}{T_{sc}},
\]
TABLE II: Modulation and coding scheme [20].

<table>
<thead>
<tr>
<th>SINR Threshold (dB)</th>
<th>-6.5</th>
<th>-6</th>
<th>-2.5</th>
<th>-1</th>
<th>1</th>
<th>2</th>
<th>6.6</th>
<th>10</th>
<th>11.4</th>
<th>11.8</th>
<th>13</th>
<th>13.8</th>
<th>15.6</th>
<th>16.8</th>
<th>17.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency (bits/symbol/Hz)</td>
<td>0.15</td>
<td>0.23</td>
<td>0.38</td>
<td>0.60</td>
<td>0.88</td>
<td>1.18</td>
<td>1.48</td>
<td>1.91</td>
<td>2.41</td>
<td>2.75</td>
<td>3.32</td>
<td>3.9</td>
<td>4.52</td>
<td>5.12</td>
<td>5.55</td>
</tr>
</tbody>
</table>

where, $S_{OFDM}$, $S_{Y_{OFDM}}$, and $T_{sc}$ represent the number of subcarriers per subchannel, number of symbols used per subcarrier, and time duration of a subframe, respectively. The factor $M$ represents number of subchannels used in downlink per sector $s$.

We consider an $\alpha$-Fair time based scheduler at each sector $s$ such that the scheduler allocates all the $M$ subchannels for a downlink time fraction denoted by $\beta_{u,s}$ to a user $u$ associated with it. In the benchmark system, we assume that any user $u$ associates with the sector $s$ from which it receives maximum received SINR on the downlink. Thus, for a user $u$, $\beta_{u,s}$ is non-zero for only one sector $s$. The resultant downlink rate for any user $u$, represented by $\lambda_u$, is given by

$$\lambda_u = \sum_{s \in S} \beta_{u,s} r_{u,s},$$

(6)

where, $r_{u,s}$ is the link rate as computed in (5). The utility function for an $\alpha$-Fair user scheduler is expressed as [21]

$$U_u(\lambda) = \begin{cases} A^{1-\alpha} & \alpha > 0, \ \alpha \neq 1, \\ 1-\alpha & \log(\lambda), \ \alpha = 1. \end{cases}$$

(7)

To focus on the downlink, we consider the TDD downlink time fraction as 1.

D. CoMP

We consider that the sectors are grouped in pre-determined CoMP clusters such that only sectors from the same CoMP cluster can cooperate and perform CoMP. This is a reasonable assumption as CoMP requires a direct backhaul link between participating sectors. Without loss of generality, we focus on the center cluster in Fig. 1 represented by $q$ such that $B_q$, $W_q$, and $\mathcal{V}_q$ denote the set of BSs, sectors, and users in the cluster $q$, respectively. Within the cluster $q$, several configurations are possible for CoMP based on which sectors perform CoMP together. We represent set of CoMP sectors present in a cluster $q$ as virtual clusters, which is represented by $\mathcal{K}_q = \{1, 2, ..., K\}$.

In a virtual cluster $k$, we use $S_k$ and $U_k$ to represent the set of sectors and users, respectively. Thus, $S_k \subseteq W_q \subset S$. We consider the following three possible CoMP configurations in the cluster $q$.

- **Configuration 1:** In this configuration, also referred to as $C_1$, as shown in Fig. (a), a CoMP user in cluster $q$ receive signals jointly from a sectors $s$ of each BS in the cluster $q$. Thus, the virtual cluster is of size $|W_q|/3$ for $C_1$.
- **Configuration 2:** In $C_2$, at most two sectors coordinate with each other as shown in Fig. (b). Thus, sectors 1, 15, and 17 do not perform CoMP, while all the other sectors perform CoMP pairwise (sectors with the same colors cooperate).
- **Configuration 3:** In Fig. (c), the Configuration 3 or $C_3$ is presented. The sectors in sets of three namely, $\{2, 9, 10\}$, $\{5, 12, 13\}$, and $\{11, 18, 19\}$ perform CoMP and the other sectors in the cluster $q$ operate without CoMP in $C_3$.

To focus on other aspects like user scheduling and resource allocation for energy saving we have considered a cluster of 7 BSs and only three CoMP configurations. However, both the cluster size and the CoMP configurations can be adapted for a practical system. The sectors present in any virtual cluster $S_k$ will vary based on the configuration under consideration as shown in Fig. 2.

We consider that the CoMP based system allocates a fraction of time for CoMP users in which the sectors in the virtual cluster transmit jointly on the downlink to the CoMP users. Whenever, the SINR of a user in the virtual cluster $S_k$ is less than a predetermined CoMP SINR threshold $\Gamma_\theta$, the user is served as a CoMP user. Let $\theta_k$ denote the time fraction in which such CoMP users receive data jointly from their virtual cluster $k$. During the remaining downlink time fraction $(1 - \theta_k)$, each sector transmits to the typical non-CoMP users individually. Note that each virtual cluster $k$ has its own $\theta_k$. 

![Various CoMP configurations for the center cluster](image-url)
In the CoMP time fraction $\theta_k$, the downlink SINR received by a user $u$ from any virtual cluster $k$ of over subchannel $m$ (denoted by $\gamma_{u,k}^m$) is given by

$$\gamma_{u,k}^m = \frac{\sum_{v \in S_k} P_{m,v} h_{m,u,v}}{\sum_{\hat{v} \in S_k} \hat{P}_{m,\hat{v}} h_{m,u,\hat{v}} + \sigma^2},$$

where, $\sum_{v \in S_k} P_{m,v} h_{m,u,v}$ is the sum of the received powers for user $u$ from all the sectors in the virtual cluster $k$ and $\sum_{\hat{v} \in S_k} \hat{P}_{m,\hat{v}} h_{m,u,\hat{v}}$ is the interference from all the other sectors in the system which are not part of this virtual cluster $k$. Note that the SINR for users associated with the non-CoMP sectors and non-CoMP users of CoMP sectors of cluster $q$ will be as in (1). The link rate for a CoMP user $u$ from a virtual cluster $k$ can be obtained using (5) and (8) as

$$r_{u,k} = \frac{\eta(\gamma_{u,k}^m) S_{C,OFDM} S_{Y,OFDM} T_{sc} M}{\tau_c}.$$

Next, we present the various BSS patterns considered in this work.

### E. BSS Patterns

Let $Z_{a1/a2}$ denote a BSS pattern in which $a1$ out of the total $a2$ BSs in the cluster are switched off. Hence, if $a1$ is equal to 0, then all BSs in the cluster are active. In Fig. 3 we depict some of the possible BSS patterns corresponding...
to $\mathbb{Z}_{1/7}$, $\mathbb{Z}_{2/7}$, $\mathbb{Z}_{3/7}$, and $\mathbb{Z}_{4/7}$ for CoMP configuration $C_1$. The shaded black triangles represent active BSs and white triangles represent the BSs that have been switched off in Fig. 3. We use idle and active states of the BSs with OFF and ON state interchangeably throughout the text. Note that Fig. 2 represents $\mathbb{Z}_0$ for $C_1$, where all BSs are active. For a given $a_1$ in $\mathbb{Z}_{a1/a2}$, multiple possible BSS patterns exist. For example, Fig. [a] and Fig. [b] are both for $\mathbb{Z}_{1/7}$. Seven such combinations are possible for $\mathbb{Z}_{1/7}$ in which any one of the seven BS in the cluster can be switched off. The proposed optimization problem and the solution heuristic are valid for all such combinations.

### F. Performance Metrics

The three key system performance metrics of a cellular network are rate, coverage, and energy. We measure the system performance of user rates through the $\alpha$-Fair throughput obtained over a cluster $q$ as follows (21)

$$T_\alpha = \left(\frac{1}{|\mathcal{V}_q|} \sum_{u \in \mathcal{V}_q} \frac{1}{\lambda_u} \right)^{\frac{1}{\alpha}}, \quad \alpha > 0, \quad \alpha \neq 1,$$

$$\left(\prod_{u \in \mathcal{V}_q} \lambda_u\right)^{\frac{1}{\alpha}}, \quad \alpha = 1, \quad (10)$$

where, $\alpha$ is the fairness parameter, $\lambda_u$ is as defined in (6), and $\mathcal{V}_q$ is the set of users associated with the cluster $q$.

We define SINR coverage as the probability of a random user $u$ receiving SINR $\gamma_{u,s}^m$ greater than the minimum SINR threshold in Table (11) from at least one sector $s$. Further, we define rate coverage as the probability of a random user $u$ receiving rate $\lambda_u$ greater than the rate threshold $\mathcal{R}$. This rate threshold is a system parameter that can be controlled by the operator.

We consider the percentage of energy saved, represented by $E$, as the metric for energy efficiency. For a given BSS pattern $\mathbb{Z}_{a1/a2}$ which means $a1$ out of $a2$ BSs are switched off, the percentage energy saving is

$$E = \frac{a1}{a2} \times 100. \quad (11)$$

Next, we consider a snapshot based approach and consider a user realization for a given user density $\mu$. We formulate the BSS as an optimization problem for this user realization.

### III. BSS Problem Formulation

We use $w_b$ as a binary BSS variable to denote BS $b$ in ON ($w_b = 0$) or OFF ($w_b = 1$) state. We focus on the cluster $q$ in the center as depicted in Fig. (1). The power consumption of a BS $b$ in idle and active state is given by $P_{idle}$ and $P_{tot}$, respectively. Then, for a given user realization, to achieve energy efficiency, we should optimize the following objective function (22)

$$\min_{w_b} \sum_{b \in B_q} w_b P_{idle}^b + (1-w_b) P_{tot}^b. \quad (12)$$

The objective function in (12) simplifies to

$$\min_{w_b} \sum_{b \in B_q} w_b P_{idle}^b - P_{tot}^b. \quad (12)$$

for a homogeneous cellular environment, is equivalent to $\max_{w_b} \sum_{b \in B_q} w_b$. Let $x_{u,s}$ denote an association variable of user $u$ with sector $s$ such that $x_{u,s} \in [0, 1]$. Then, the BSS problem can be framed as an optimization problem for a given user realization as follows.

$$\mathbb{B} : \max_{w_b} \sum_{b \in B_q} w_b \quad (13)$$

s.t. $\sum_{b \in B_q} w_b \leq |B_q| - 1, \quad (14)$

$$w_b \in [0, 1], \quad \forall b \in B_q, \quad (15)$$

$$x_{u,s} = \begin{cases} 1, & \text{if } s = \arg \max_s (\gamma_{u,s}^m) \\ 0, & \text{otherwise} \end{cases}, \quad \forall u \in \mathcal{V}_q, \forall s \in \mathcal{W}_q, \quad (16)$$

$$\gamma_{u,s}^m = \sum_{s \in S} (1-w_{[s/3]})P_{idle}^m \gamma_{u,s}^m + \alpha^2 \quad (17)$$

$$\lambda_u = \sum_{s \in \mathcal{W}_q} \beta_{u,s} x_{u,s} \lambda_u \quad \geq \mathcal{R}, \quad \forall u \in \mathcal{V}_q, \quad (18)$$

where, the constraint (14) implies that at least one BS in the cluster $q$ should be in the ON state, (15) reflects that a BS can be either in ON or OFF state, (16) is required as the user should associate to a sector $s$ from which it receives maximum SINR, the modified SINR from a sector $s$ in (17) is required as the received power from a sector $s$ corresponding to BS $b = [s/3]$ or received power from an interfering sector $s$ can be zero if the corresponding BS is switched off, and (18) guarantees that BSS should not result in any user’s rate being less than the rate threshold $\mathcal{R}$. Next, we formulate the CoMP based optimization problem for a virtual cluster $k$ that is part of the center cluster $q$.

### IV. CoMP Problem Formulation

For the CoMP based system, we use $z_{u,s}$ as a binary variable that denotes whether the user $u$ associated to sector $s$ will receive CoMP transmission from the virtual cluster $k$ (such that $s \in S_k$ and $z_{u,s}=1$) or will receive conventional downlink transmission from the sector $s$ ($z_{u,s}=0$). We set the value of $z_{u,s}$ as 1 if the $\gamma_{u,s}^m$ is less than the CoMP SINR threshold $\Gamma_d$. Given the number of CoMP and non-CoMP users, the virtual cluster $k$ has to decide the optimal CoMP time fraction $\theta_k$. Further, we define $\beta_{u,k}$ as the time fraction of $\theta_k$ for which an individual CoMP user $u$ receives joint downlink transmission from the virtual cluster $k$. Then, given the utility function in (7), the joint CoMP resource allocation and user scheduling problem for a virtual cluster $k$ can be formulated as the following optimization problem.

$$\mathbb{P} : \max_{\Lambda_k, \beta_{u,k}} \sum_{u \in S_k} U_u(\lambda_u), \quad (19)$$

s.t. $\lambda_u = (1-\theta_k) \sum_{s \in S_k} x_{u,s}(1-z_{u,s})\beta_{u,s} x_{u,s} \lambda_u + \theta_k \sum_{s \in S_k} x_{u,s} z_{u,s} \beta_{u,s} x_{u,s}, \quad \forall u \in S_k, \quad (20)$
\[ x_{u,s} = \begin{cases} 1, & \text{if } s = \arg \max \{ y_{u,s}^m \}, \\ 0, & \text{otherwise}, \forall u \in \mathcal{U}_k, \forall s \in S_k, \end{cases} \tag{21} \]

\[ z_{u,s} = \begin{cases} 1, & \text{if } y_{u,s}^m \leq \Gamma_d x_{u,s}, \ s \in S_k \ s.t. |S_k| > 1, \\ 0, & \text{otherwise}, \forall u \in \mathcal{U}_k, \forall s \in S_k, \end{cases} \tag{22} \]

\[ \sum_{u \in \mathcal{U}_k} \sum_{s \in S_k} z_{u,s} x_{u,s} \beta_{u,k} \leq 1, \tag{23} \]

\[ \sum_{u \in \mathcal{U}_k} (1 - z_{u,s}) x_{u,s} \beta_{u,k} \leq 1, \forall s \in S_k, \tag{24} \]

\[ \beta_{u,k} \geq 0, \forall u \in \mathcal{U}_k, \forall s \in S_k, \tag{25} \]

\[ \beta_{h,k} \geq 0, \forall u \in \mathcal{U}_k, \tag{26} \]

\[ \theta_k \in [0, 1], \tag{27} \]

\[ \Gamma_d \in \left[ \xi_{\text{min}}^d, \xi_{\text{max}}^d \right], \tag{28} \]

where, the user rate is defined by \((26)\) such that any non-CoMP users \(u\) gets a fraction of \(\beta_{u,k}(1 - \theta_k)\) from the sector \(s\) and any CoMP users \(u\) gets a fraction of \(\beta_{u,k}\theta_k\) from all sectors in \(k\). \(x_{u,s}\), in \((21)\) represents the maximum SINR based binary user association variable, the constraint in \((22)\) implies that a user can be either CoMP or non-CoMP with corresponding binary \(z_{u,s}\), \((23)\) indicates that time fractions of \(\theta_k\) allocated to all CoMP users in cluster \(k\) must be less than equal to 1. Similarly, \((24)\) indicates that time fractions of \((1 - \theta_k)\) allocated individually in each sector \(s\) to non-CoMP users should be less than equal to 1. The constraints in \((25)\) and \((26)\) are required to ensure non-negative time fractions. The constraint in \((27)\) ensures that CoMP time fraction is not more than the total available time. The values of \(\xi_{\text{min}}^d\) and \(\xi_{\text{max}}^d\) in \((28)\) define the permitted range for the CoMP threshold \(\Gamma_d\). The \(r_{u,k}\) in \((20)\) is given in \((2)\).

The joint resource allocation and user scheduling problem in \((19)\) is a mixed integer non-linear program (MINLP) which is difficult to solve in general for the multiple optimization variables simultaneously (namely, \(\Gamma_d\), \(\theta_k\), \(\beta_{u,s}\), \(\beta_{h,k}\)). Hence, we next present propositions that provide individual optimal solutions with respect to \(\beta_{u,s}\), \(\beta_{h,k}\), and \(\theta_k\) for a given \(\Gamma_d\) and \(x_{u,s}\) in a virtual cluster \(k\).

**Proposition 1.** For a virtual cluster \(k\), given a user association \(x_{u,s}\), a CoMP SINR threshold \(\Gamma_d\), at least one CoMP user with \(\gamma_{u,s}^m \leq \Gamma_d\), and any CoMP time fraction \(\theta_k\), the optimal time fraction of \((1 - \theta_k)\), allocated by the \(\alpha\)-Fair scheduler in any sector \(s \in S_k\) for a non.CoMP user \(u\) is equal to

\[ \beta_{u,s}^* = \frac{\tau_{u,s}}{\sum_{s \in S_k} \tau_{u,s}}, \forall s \in S_k, \forall u \in \mathcal{U}_c, \tag{29} \]

where, \(\tau_{u,s}\) = \(r_{u,s}\), and the optimal time fraction of \(\theta_k\) allocated by an \(\alpha\)-Fair scheduler for all the sectors jointly to a CoMP user \(u\) is equal to

\[ \beta_{h,k}^* = \frac{\tau_{u,k}}{\sum_{u \in \mathcal{U}_c} \tau_{u,k}}, \forall u \in \mathcal{U}_c, \tag{30} \]

where, \(\tau_{u,k}\) = \(r_{u,k}\), \(\mathcal{U}_c = \{1, 2, ... U_c\}\), \(\mathcal{U}_nc = \{1, 2, ... U_{nc}\}\), and \(\mathcal{U}_{nc,s} = \{1, 2, ... U_{nc,s}\}\) denote the set of CoMP users in \(S_k\), the set of non-CoMP users in \(S_k\), and the set of non-CoMP users in any sector \(s \in S_k\) in the virtual cluster, respectively.

**Proof:** For any given user association \(x_{u,s}\) (note that it need not be maximum SINR based) and CoMP SINR threshold \(\Gamma_d\), the virtual cluster \(k\) can compute \(z_{u,s}\) using \((22)\). Given binary \(z_{u,s}\), a user \(u\) can be classified as CoMP or non-CoMP user into the sets \(\mathcal{U}_c\) or \(\mathcal{U}_nc\), respectively. Further, the set of non-CoMP users for every sector \(s \in S_k\), denoted by \(\mathcal{U}_{nc,s}\), can be obtained. Then, as \(\mathcal{U}_c = \mathcal{U}_c \cup \mathcal{U}_{nc}\), the objective function in \((19)\) denoted by \(Y\) can be represented as

\[ Y = \sum_{u \in \mathcal{U}_c} \sum_{s \in S_k} \left(1 - \theta_k\right) \left(1 - \alpha\right) \beta_{u,s}^* + \sum_{u \in \mathcal{U}_c} \sum_{s \in S_k} \theta_k \beta_{h,k}^* = \sum_{u \in \mathcal{U}_c} \sum_{s \in S_k} \left(1 - \theta_k\right) \left(1 - \alpha\right) \beta_{u,s}^* + \sum_{u \in \mathcal{U}_c} \sum_{s \in S_k} \theta_k \beta_{h,k}^*, \tag{31} \]

which using \((20)\) becomes

\[ Y = \sum_{u \in \mathcal{U}_c} \sum_{s \in S_k} \left(1 - \theta_k\right) \left(1 - \alpha\right) \beta_{u,s}^* + \sum_{u \in \mathcal{U}_c} \sum_{s \in S_k} \theta_k \beta_{h,k}^* \tag{32} \]

Then, for any given \(\theta_k\), \(x_{u,s}\), and \(\Gamma_d\), the optimization problem in \((19)\) can be simplified to

\[ \mathbb{P}^* : \max_{\beta_{u,s}, \beta_{h,k}, \theta_k} Y \tag{33} \]

s.t.

\[ \sum_{u \in \mathcal{U}_c} \sum_{s \in S_k} \beta_{u,s} \leq 1, \forall s \in S_k, \tag{34} \]

\[ \sum_{u \in \mathcal{U}_c} \sum_{s \in S_k} \beta_{h,k} \leq 1, \tag{35} \]

where, \((33)\) and \((34)\) are obtained from \((23)\) and \((24)\), respectively. The Lagrangian function of \((32)\) can be defined as

\[ \mathcal{L}(Y, \mathcal{V}_s, \mathcal{V}_k, \mathcal{X}_{u,s}, \mathcal{X}_{u,k}) = -Y + \sum_{u \in \mathcal{U}_c} \sum_{s \in S_k} \mathcal{V}_s \left( \sum_{u \in \mathcal{U}_c} \beta_{u,s} - 1 \right) + \sum_{u \in \mathcal{U}_c} \sum_{s \in S_k} \mathcal{V}_k \left( \sum_{u \in \mathcal{U}_c} \beta_{h,k} - 1 \right). \tag{36} \]

The first-order stationarity conditions of \((36)\) for \((33)\) and \((34)\) result in

\[ \frac{d\mathcal{L}}{d\mathcal{V}_s} = - \left(1 - \theta_k\right) r_{u,s} \left(1 - \alpha\right) \beta_{u,s}^* + \mathcal{V}_s = 0 \tag{37} \]

\[ \frac{d\mathcal{L}}{d\mathcal{V}_k} = - \theta_k r_{u,k} \left(1 - \alpha\right) \beta_{h,k}^* + \mathcal{V}_k = 0, \tag{38} \]

Solving \((37)\) and \((38)\) jointly with \((33)\) and \((34)\) result in \((29)\) and \((30)\), respectively. This completes the proof of Proposition 1.
Note that for \( \alpha = 1 \), i.e., a proportional fair scheduler, \([29]\) and \([30]\) result in time fractions \(1/N_{nc,t} \) and \(1/N_c\) for Non-CoMP and CoMP users, respectively, in any sector \( s \) of the CoMP cluster.

**Proposition 2.** For a given user association \( x_{u,s} \) and CoMP SINR threshold \( \Gamma_d \), the optimal time fraction \( \theta^*_k \) for CoMP users in a virtual cluster \( k \) is given by

\[
\theta^*_k = \frac{\delta}{1 + \delta},
\]

where,

\[
\delta = \left[ \frac{\sum_{u \in U} (r_{u,k}\beta_{u,k})^{1-\alpha}}{\sum_{u \in U, s \in S_k} x_{u,s}(r_{u,k}\beta_{u,k})^{1-\alpha}} \right] \frac{1}{\alpha},
\]

with \( \beta_{u,s} \) and \( \beta_{u,k} \) as in \([29]\) and \([30]\), respectively.

**Proof:** For any given user association \( x_{u,s} \) and CoMP SINR threshold \( \Gamma_d \), the virtual cluster \( k \) may be classified users into the sets \( U_c \) or \( U_{nc} \) as shown in the proof of Proposition 1. Then, as \( U_k = U_c \cup U_{nc} \), the objective function in \([19]\) can be represented as

\[
\sum_{u \in U_k} \frac{1}{\alpha} = \sum_{u \in U_c} \frac{1}{1-\alpha} + \sum_{u \in U_{nc}} \frac{1}{1-\alpha},
\]

which gives \( x_{u,s} \) binary, \([20]\), \([29]\), and \([30]\) becomes

\[
\sum_{u \in U_c} \sum_{s \in S_k} x_{u,s}(1-\theta_k)^{1-\alpha}(r_{u,k}\beta_{u,k})^{1-\alpha} + \sum_{u \in U_{nc}} \frac{1}{\alpha} \sum_{u \in U_c} (r_{u,k}\beta_{u,k})^{1-\alpha}.
\]

Differentiating \([41]\) with respect to \( \theta_k \) and equating to 0 gives

\[
(1 - \theta_k)^{-\alpha} \sum_{u \in U_{nc}, s \in S_k} x_{u,s}(r_{u,k}\beta_{u,k})^{1-\alpha} = (\theta_k)^{-\alpha} \sum_{u \in U_{nc}} (r_{u,k}\beta_{u,k})^{1-\alpha},
\]

which on simplification results in \([39]\). This completes the proof of Proposition 2.

The result presented in \([39]\) is valid for any \( \alpha \)-Fair scheduler. The optimal CoMP time fraction \( \theta^*_k \) for some commonly used \( \alpha \)-Fair schedulers is presented in Table \( \text{III} \). Note that for a proportional fair scheduler \( \alpha = 1 \), \( \theta^*_k \) is independent of the user link rates and the time allocated to each user. In Table \( \text{III} \) the \( N_{nc} \) and \( N_c \) in a virtual cluster \( k \) are given by

\[
N_{nc} = \sum_{u \in U_{nc}} (1 - z_{u,s})x_{u,s}, \quad \text{and}
\]

\[
N_c = \sum_{u \in U_c} z_{u,s}x_{u,s}, \quad \text{respectively.}
\]

Given the BSS optimization problem for center cluster \( q \) in \([13]\) and the CoMP optimization problem in \([19]\) for any virtual cluster \( k \) that is a part of the center cluster \( q \), we next formulate the joint BSS with CoMP as an optimization problem over the center cluster \( q \).

**V. BSS with CoMP Problem Formulation**

Given a CoMP SINR threshold \( \Gamma_d \), we consider a maximum SINR based user association. Further, any user in the center cluster \( q \) should obtain a rate higher than a pre-determined rate threshold \( R \) with or without CoMP from corresponding virtual cluster \( k \) or sector \( s \), respectively. Then, the BSS with CoMP can be formulated as an optimization problem for the center cluster \( q \) as follows.

\[
\mathbb{B}^* : \max_{w_b} \sum_{b \in B} w_b
\]

s.t. \([14],[17]\),

\[
\lambda_u = \left[ \sum_{k \in K_q} \left(1 - \theta^*_k\right)x_{u,s}(1 - z_{u,s})\beta_{u,k}^\alpha r_{u,s}\right] + \sum_{k \in K_q} \sum_{s \in S_k} \theta^*_k x_{u,s} z_{u,s} \beta_{u,k}^\alpha r_{u,k} > \mathcal{R} \ \forall u \in V_q,
\]

\[
\gamma^m_{u,k} = \sum_{s \in S_k} \left(1 - w_{\forall s}(\beta_{u,k}^\alpha r_{u,k})^m + \sigma^m\right),
\]

\[
x_{u,s} = \begin{cases} 1, & \text{if } s = \arg\max_{s \in S} \{\gamma^m_{u,s}\}, \\ 0, & \text{otherwise, } \forall u \in V_q, \forall s \in W_q, \end{cases}
\]

\[
z_{u,s} = \begin{cases} 1, & \text{if } \gamma^m_{u,s} \leq \Gamma_d x_{u,s}, s \in S_k, k \in K_q \text{ s.t. } |S_k| > 1, \\ 0, & \text{otherwise, } \forall u \in V_q, \forall s \in W_q, \end{cases}
\]

\[
\beta_{u,s}^\alpha \text{ is as in } [29], \forall u \in V_q, \forall s \in W_q,
\]

\[
\beta_{u,k}^\alpha \text{ is as in } [30], \forall k \in K_q,
\]

\[
\theta^*_k \text{ is as in } [39], \forall k \in K_q,
\]

where, the objective function in \([44]\) is the same as in \([13]\), constraint \([14]\) is to ensure that at least one BS is in the center cluster is in ON state, constraint \([17]\) is required to account for the change in SINR from a sector with BSS, \([45]\) is the resultant rate of a user with BSS and CoMP, the SINR from virtual cluster \( k \) is recomputed in \([46]\) as with BSS the received power from a sector \( v \) corresponding to BS \( b = \lceil v/3 \rceil \) or received power from an interfering sector \( \hat{v} \) can be zero if the corresponding BS is switched off. \([47]\) is required to re-compute user association with BSS through the additional term of \((1-w_b)\) that ensures the maximum SINR is computed only over the BSs that are still in ON state, the constraint in \([48]\) ensures that a user is served as a CoMP user based on received SINR only from sectors of BSs still in ON state and for virtual cluster with more than one sector available for CoMP, \( \beta_{u,s}^\alpha, \beta_{u,k}^\alpha, \theta^*_k \) in \([29],[30],[39]\) have to be
Algorithm 1 Dynamic Base Station Switching with CoMP

1. INPUTS : \{P_{s}^{m}h_{u,s}^{m}\}, \mathcal{V}_{q}, \Gamma_{d}, R, \{Z_{a_1/a_2}\}
2. OUTPUTS : \{\lambda_{u}\}, Z_{a_1/a_2}
3. Sort $Z_{a_1/a_2}$ in increasing order of energy consumption
4. Initialize : $J = ||Z_{a_1/a_2}||$, $j = 1$
5. **Repeat**
6. Initialize : $u = 1$, \{za,u\} = 0
7. **Repeat**
8. Sort \{P_{s}^{m}h_{u,s}^{m}\} in decreasing order and set $x_{a,u} = 1$
9. $\gamma_{u,s} = f(P_{s}^{m}h_{u,s}^{m})$ as in (17)
10. if $\gamma_{u,s} \leq \Gamma_{d}$ then
11. $\gamma_{u,k} = f(P_{s}^{m}h_{u,s}^{m})$ as in (46)
12. $z_{a,u} = 1$
13. else
14. $z_{a,u} = 0$
15. end if
16. $\setminus u = u + 1$
17. **Until** $u \geq \mathcal{V}_{q}$ + 1
18. Set $u = 1$
19. **Repeat**
20. Compute $\lambda_{u}$ as in (45)
21. Set $u = u + 1$
22. **Until** $u \geq \mathcal{V}_{q}$ + 1
23. if min(\{\lambda_{u}\}) < R and $j < J$ then
24. $j = j + 1$
25. **Goto** Step. 6
26. else
27. $Z_{a_1/a_2} = Z_{a_1/a_2}^{j}$
28. **Goto** Step. 31
29. end if
30. **Until** $j > J$
31. Stop

TABLE IV: Simulation Parameters

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>$B$</strong></td>
<td>49</td>
</tr>
<tr>
<td><strong>Inter-site Distance</strong></td>
<td>500 m</td>
</tr>
<tr>
<td><strong>Penetration loss($\mu$)</strong></td>
<td>20 dB</td>
</tr>
<tr>
<td><strong>Slow Fading($\rho$)</strong></td>
<td>Standard deviation of 8 dB</td>
</tr>
<tr>
<td>$P_{BS}$</td>
<td>46 dBm</td>
</tr>
<tr>
<td><strong>$n^2$</strong></td>
<td>2.2661e-15</td>
</tr>
<tr>
<td><strong>PL(d)</strong></td>
<td>136.8245+(39.0866log_{10}(d-3)) [19]</td>
</tr>
<tr>
<td><strong>$M$</strong></td>
<td>99</td>
</tr>
<tr>
<td><strong>Subchannel Bandwidth</strong></td>
<td>180 KHz</td>
</tr>
<tr>
<td><strong>ScCPDM</strong></td>
<td>12</td>
</tr>
<tr>
<td><strong>SYCPDM</strong></td>
<td>14</td>
</tr>
<tr>
<td><strong>Subframe</strong></td>
<td>1 ms</td>
</tr>
<tr>
<td><strong>Cluster size</strong></td>
<td>7</td>
</tr>
</tbody>
</table>

**VI. PROPOSED HEURISTIC FOR BSS WITH COMP**

In this section, we present a heuristic that selects the optimum BSS pattern for a pre-determined set of virtual clusters that perform CoMP in the center cluster $q$. The proposed heuristic assumes that the set of users \mathcal{V}_{q} and the set of received powers for any user $u$ from any sector $s$, represented by \{P_{s}^{m}h_{u,s}^{m}\}, is available. The heuristic considers a set of BSS patterns denoted by \{Z_{a_1/a_2}\}. Note that any element $Z_{a_1/a_2}$ of this set is equivalent to a unique combination of $w_{b}$, the binary BSS indicator variables specified in (15). The heuristic also takes $\Gamma_{d}$ and $R$ as an input. The set of BSS patterns is first sorted in an increasing order of energy consumption such that any BSS pattern \{Z_{a_1/a_2}\} consumes less than equal to the energy consumed by \{Z_{a_1/a_2}^{j}\}. The heuristic starts with least energy consuming BSS pattern. Next, the set of received powers \{P_{s}^{m}h_{u,s}^{m}\} is sorted for any user $u$ from all sectors $s$. Using this operation for every user $u$, the sector $s$ from which it receives maximum power is identified and $x_{a,u}$ is set as 1. Next, given $R$ it is decided whether a user $u$ is a CoMP or a non-CoMP user. Then, for the BSS pattern under consideration, the received SINRs from the corresponding sector or virtual cluster is computed using (17) or (46), respectively. Note that (17) and (46) consider only the BSs that are still in ON state for the SINR calculations. In a separate loop over the number of users, i.e., $\mathcal{V}_{q}$, the rate of each user is computed. This is required as the user association and SINRs are used to compute the rate of all users in the system as in (45). In case all users receive a rate higher than the rate threshold $R$ then the heuristic stops and selects this BSS pattern as the optimum pattern. Otherwise, the number of switched on BSs is increased and the described steps are repeated for the next BSS pattern. The heuristic runs till either a optimum BSS pattern is obtained or all BSs are in ON state.

The heuristic is presented as a pseudo-code in Algo. 1. The complexity of the proposed heuristic for every user realization is $O(J(\mathcal{V}_{q}\|B_{q}\| + \mathcal{V}_{q}))$. Note that worst case $J$ is equal to $2^{B_{q}}$. However, in practice, operators can optimize and choose from a lower number of BSS patterns. For example, in the numerical results presented next, we consider $J$ equal to five BSS patterns.

**VII. NUMERICAL RESULTS**

We consider a center cluster with 7 BSs. To model the interference suitably, we consider a wrap-around system with 6 clusters of 7 BS each around the center cluster. We consider the simulation parameters specified by 3GPP for an urban homogeneous cellular environment as given in [19]. Thus, a total of 49 BSs are considered for simulations with inter-site distance of 500 m. The users are distributed uniformly randomly with the appropriate user density ($\mu$) over the entire simulations area. We consider 500 user location realizations. For each location realization the results are averaged over 50 independent fading realizations. The simulation parameter details are given in Table IV. To study the impact of change in $\mu$ over the system performance, we vary the average user density from 20 to 160 users per $Km^2$.

The variation of $\theta_{k}$ with respect to $\Gamma_{d}$ is shown in Fig. 4 for various values of $\alpha$. Note that the optimal value of $\theta_{k}$ obtained via exhaustive search in simulations matches with the $\theta_{k}$ derived in (39). Further, the optimal CoMP time fraction increases with an increase in the CoMP SINR threshold as

computed using (29), (30), and (39), respectively. Note that the optimization problem presented in (44) is a MINLP and the problem becomes complex for large number of BSSs, i.e., $|B_{q}|$. Next, we present a heuristic that solves the joint BSS with CoMP optimization problem.
more number of users become CoMP users with increase in $\Gamma_d$. The increase in $\alpha$ values makes the $\alpha$-Fair scheduler allocate more resources to edge users. Hence, an increase in the fairness parameter $\alpha$ results in an increase in $\theta^*_k$ for the same value of $\Gamma_d$. The increased $\theta^*_k$ ensures that the edge users (with SINR $\leq \Gamma_d$) will be served as CoMP users and receive more downlink time fraction.

The throughput metric corresponding to a $\alpha$–Fair scheduler is given in (10). In Fig. 5 the variation of the throughput metric ($T_{\alpha}$) is presented with respect to CoMP SINR threshold ($\Gamma_d$), with and without CoMP, for various $\alpha$’s and CoMP configurations. Note that from a rate perspective the overall system performs better without CoMP in comparison with various CoMP configurations like $C_1$, $C_2$, and $C_3$ because in CoMP resources of more than one sectors are used to improve SINR of CoMP users. Thus, the overall system throughput is bounded by benchmark without CoMP system throughput. Further, Fig. 5 shows that a lower value of $\Gamma_d$ will result in less loss in system throughput.

The probability of coverage (for SINR coverage as defined in Section II F) is presented for $Z_0$ (all BSs in ON state) and BSS patterns $Z_{1/7}$, $Z_{2/7}$, $Z_{3/7}$ and $Z_{4/7}$ for various modes of CoMP operations are shown in Fig. 6. Note that the SINR coverage increases for all the CoMP configurations in comparison to without CoMP scenario. Further, an increase in the number of switched off BSs results in decrease in the coverage probability for all CoMP configurations. The results considered are for BSS patterns shown in Fig. 3b, 3d, 3h, and 3i.

The variation of $T_{\alpha}$ with respect to $\Gamma_d$, different BSS patterns, and $\alpha = 1$ is presented in Fig. 7. Note that the throughput decreases as more BSs are switched off. Further, even with various BSS patterns, the without CoMP scenario, CoMP configuration $C_3$, $C_2$, and $C_1$ are in decreasing order of throughput. This is due to the rate and coverage trade-off between these configurations. For example, in Fig. 6 the coverage probability of $C_1$ is higher than $C_2$ for all BSS scenarios. Whereas, in Fig. 7 the throughput of $C_1$ is lower than $C_2$ for all BSS scenarios. Thus, multiple configurations.
of CoMP with BSS can be used to achieve various trade-offs between rate and coverage which a traditional without CoMP system does not offer.

For the rest of the results, we focus on $C_3$ as it results in least loss in throughput in comparison to without CoMP scenario. The rate coverage as defined in Section IIF is presented in Fig. 8 and Fig. 9 for $\alpha = 1$ and $\Gamma_d = -1 \text{ dB}$. In Fig. 8 the probability to operate in with a BSS pattern while ensuring the user rates to be higher than the rate threshold $R$ is presented for without CoMP and with CoMP configuration $C_3$ scenarios is presented. Fig. 8 shows that to maintain the same rate coverage with large energy savings the system has to reduce the rate threshold. Further, for the same rate threshold, BSS patterns with higher energy savings are less probable. Note that Fig. 8 is for BSS pattern $Z_{2/7}$. It is observed from Fig. 9 that the probability for selecting the BSS pattern increases with increase in $\alpha$.

The edge user rates calculated for the lowest ten percentile users in the system are presented in Fig. 10. Note that as the user density reduces, the various BSS patterns along with CoMP configurations offer a much higher granularity of operating points that an operator can select. Thus, using the work presented in this paper an operator can select the right rate, coverage, and energy trade-off. For example, whenever the user density in Fig. 10 drops from 100 to 40 per km$^2$, an operator can switch from without CoMP $Z_0$ to without CoMP any other BSS patterns to save energy at the cost of SINR coverage as shown in Fig. 6. Alternatively, the operator can use, one of the CoMP configurations to maintain the same edge user rates, without compromising on the SINR coverage.

In Fig. 11 the result from the heuristic proposed in Section VI is presented. We select $R$ as 0.2Mbps. A snapshot of traffic profile variation is selected and an optimum BSS pattern ($Z_j$) is selected based on the given operator rate threshold $R$. In Fig. 11 a1 represents the number of BSs switched off and correspondingly the percentage energy saved. It is observed from Fig. 11 that there is some decrease in overall
throughput whenever BSs are switched off. However, the loss in throughput is accompanied with significant gain in terms of energy savings. Thus, the proposed heuristic ensures maximum energy savings, without loss in coverage, at the cost of high rate users.

VIII. CONCLUSION

We have shown that loss in SINR coverage due to BSS can be compensated by CoMP transmission. We have formulated the CoMP user scheduling and resource allocation as an optimization problem. Optimal solutions for user scheduling and CoMP time fractions have been derived. The derived results have been used to formulate and solve the challenging problem of BSS with CoMP. A heuristic has been presented that solves the BSS with CoMP problem dynamically. Through numerical results it has been shown that the derived results match closely with simulations. Further, we have shown that BSS with CoMP can be used to achieve various possible trade-offs in energy savings, throughput, and coverage. In future, the presented work will be extended using a stochastic geometry based framework for arbitrary cluster size.

REFERENCES