I. INTRODUCTION

The rare decays of $B$ mesons involving flavor changing neutral current (FCNC) transitions are of great interest to look for possible hints of new physics beyond the standard model (SM). In the SM, the FCNC transitions arise only at one-loop level, thus providing an excellent testing ground to look for new physics. Therefore, it is very important to study FCNC processes, both theoretically and experimentally, as these decays can provide a sensitive test for the investigation of the gauge structure of the SM at the loop level. Huge experimental data on both exclusive and inclusive $B$ meson decays [1] involving $b \to s$ transitions have been accumulated at the $e^+e^-$ asymmetric $B$ factories operating at Y(4S), which motivated extensive theoretical studies on these mesonic decay modes.

Unlike the mesonic decays, the experimental results on FCNC mediated $\Lambda_b$ baryon decays e.g., $\Lambda_b \to \Lambda \pi$, $\Lambda_b \to p K^-$, $\Lambda_b \to \Lambda \gamma$, and $\Lambda_b \to \Lambda l^+l^-$ are rather limited. At present we have only upper limits on some of these decay modes [2]. Heavy baryons containing a heavy $b$ quark will be copiously produced at the LHC. Their weak decays may provide important clues on flavor changing currents beyond the SM in a complementary fashion to the $B$ decays. A particular advantage of the bottom baryon decays over the $B$ mesons is that these decays are self-tagging processes which should make their experimental reconstructions easier.

Another important aspect is that, in the past few years, we have seen some kind of deviations from the SM results in the $CP$ violating observables of $B$ and $B_s$ meson decays involving $b \to s$ transitions [1,3–6]. Several new physics scenarios are proposed in the literature to account for these deviations [7]. Therefore, it is quite natural to expect that if there is some new physics present in the $b \to s$ transitions of $B$ meson decays it must also affect the corresponding $\Lambda_b$ transitions. Therefore, the study of the rare $\Lambda_b$ decays is of utmost importance to obtain an unambiguous signal of new physics.

In this paper we would like to study the rare $\Lambda_b$ decays in a model with an extra generation of quarks, usually known as SM4 [8]. SM4 is a simple extension of the standard model with three generations (SM3) with the additional up-type ($t'$) and down-type ($d'$) quarks. The model retains all the properties of SM3. The $t'$ quark like the other up-type quarks contributes to the $b \to s$ transition at the loop level. Because of the additional fourth generation there will be mixing between the $b'$ quark, the three down-type quarks of the standard model, and the resulting mixing matrix will become a $4 \times 4$ matrix ($V_{\text{CKM}4}$). The parametrization of this unitary matrix requires six mixing angles and three phases. The existence of the two extra phases provides the possibilities of an extra source of $CP$ violation. Another advantage of this model is that the heavier quarks and leptons in this family can play a crucial role in dynamical electroweak symmetry breaking as an economical way to address the hierarchy problem [9]. The effects of the fourth generation of quarks in various $B$ decays are extensively studied in the literature [10]. In Refs. [11,12], it has been shown that this model can easily explain the observed anomalies in the $B$ meson sector.

The paper is organized as follows. In Sec. II we discuss the nonleptonic decay of the $\Lambda_b$ baryon. The radiative decay process $\Lambda_b \to \Lambda \gamma$ is discussed in Sec. III. The results on semileptonic decays are presented in Sec. IV. Section V contains the summary and conclusion.

II. DECAY WIDTH OF $\Lambda_b \to \Lambda \pi^0$ AND $\Lambda_b \to p K^-$ MODES

In this section we will discuss the nonleptonic rare $\Lambda_b$ decay modes $\Lambda_b \to \Lambda \pi$ and $\Lambda_b \to p K^-$ induced by the quark level transition $b \to s q \bar{q}$ ($q = u, d$). The effective Hamiltonian describing these processes is given by [13]

$$\mathcal{H}_\text{eff} = \frac{G_F}{\sqrt{2}} \left[ V_{ub}V_{us}^* \sum_{i=1,2} C_i(\mu)O_i - V_{tb}V_{ts}^* \sum_{i=3}^{10} C_i(\mu)O_i \right],$$

(1)
where $C_i(\mu)$'s are the Wilson coefficients evaluated at the renormalization scale $\mu$. $O_{1,2}$ are the tree level current-current operators, $O_{3-6}$ are the QCD and $O_{7-10}$ are the electroweak penguin operators.

Let us first consider the decay process $\Lambda_b \rightarrow \Lambda \pi$. In the SM this mode receives contributions from the color-suppressed tree and the electroweak penguin diagrams and the amplitude for this process in the factorization approximation is given as \cite{[14]}

$$A(\Lambda_b(p) \rightarrow \Lambda(p')\pi^0(q)) = \frac{G_F}{\sqrt{2}} \left[ V_{ub}V_{us}^*a_2 - \frac{3}{2} V_{tb}V_{ts}^*(a_9 - a_7) \right] \times \langle \Lambda(p')|\bar{u}\gamma^\mu(1 - \gamma_5)b|\Lambda_b(p)\rangle \times \langle \pi^0(q)|\bar{u}\gamma_\mu(1 - \gamma_5)u|0\rangle,$$

where $a_i = C_i + C_{i+1}/N(C_i + C_{i-1}/N)$ for $i =$ odd (even). In order to evaluate the matrix elements we use the following form factors and decay constants. The matrix elements of the various hadronic currents between initial $\Lambda_b$ and the final $\Lambda$ baryon are parametrized in terms of various form factors \cite{[15]} as

$$\langle \Lambda(p')|\bar{u}\gamma_\mu b|\Lambda_b(p)\rangle = \bar{u}_A(p')[g_1(q^2)\gamma_\mu + ig_2(q^2)\gamma_\mu\gamma_5 + g_3(q^2)\gamma_\mu q_\mu u_{\Lambda_b}(p),$n

$$\langle \Lambda(p')|\bar{u}\gamma_\mu\gamma_5 b|\Lambda_b(p)\rangle = \bar{u}_A(p')[G_1(q^2)\gamma_\mu + ig_2(q^2)\gamma_\mu\gamma_5 + g_3(q^2)\gamma_\mu q_\mu ]\gamma_5 u_{\Lambda_b}(p),$$

where $g_i$ $(G_i)$'s are the vector (axial vector) form factors and $q$ is the momentum transfer i.e., $q = p - p'$. The matrix element $\langle \pi^0(q)|\bar{u}\gamma_\mu\gamma_5 u|0\rangle$ is related to the pion decay constant $f_\pi$ as

$$\langle \pi^0(q)|\bar{u}\gamma_\mu\gamma_5 u|0\rangle = if_\pi q^\mu/\sqrt{2}.$$ (4)

With these values one can write the transition amplitude for $\Lambda_b \rightarrow \Lambda \pi$ as

$$A(\Lambda_b \rightarrow \Lambda \pi^0) = i \frac{G_F}{2} f_\pi \left[ V_{ub}V_{us}^*a_2 - \frac{3}{2} V_{tb}V_{ts}^*(a_9 - a_7) \right] \times \bar{u}_A(p')[G_1(q^2)(m_{\Lambda_b} - m_\Lambda) + g_3(q^2)m_\pi^2]u_{\Lambda_b}(p) + (G_1(q^2)(m_{\Lambda_b} + m_\Lambda) - G_3(q^2)m_\pi^2)]\gamma_5 u_{\Lambda_b}(p).$$ (5)

The above amplitude can be symbolically written as

$$A(\Lambda_b(p') \rightarrow \Lambda(p)\pi^0(q)) = i\bar{u}_A(p')(A + B\gamma_5)u_{\Lambda_b}(p),$$

where $A$ and $B$ are given as

$$A = \frac{G_F}{2} f_\pi \left[ V_{ub}V_{us}^*a_2 - \frac{3}{2} V_{tb}V_{ts}^*(a_9 - a_7) \right] \times (g_1(q^2)(m_{\Lambda_b} - m_\Lambda) + g_3(q^2)m_\pi^2),$$

$$B = \frac{G_F}{2} f_\pi \left[ V_{ub}V_{us}^*a_2 - \frac{3}{2} V_{tb}V_{ts}^*(a_9 - a_7) \right] \times (G_1(q^2)(m_{\Lambda_b} + m_\Lambda) - G_3(q^2)m_\pi^2).$$ (7)

Thus, one can obtain the decay width for this process as \cite{[16]}

$$\Gamma = \frac{p_{c.m.}}{8\pi} \left[ \frac{(m_{\Lambda_b} + m_\Lambda)^2 - m_\pi^2}{m_{\Lambda_b}^2} |A|^2 + \frac{(m_{\Lambda_b} - m_\Lambda)^2 - m_\pi^2}{m_{\Lambda_b}^2} |B|^2 \right],$$ (8)

where $p_{c.m.}$ is the magnitude of the center-of-mass momentum of the outgoing particles.

For numerical analysis we use the following input parameters. The masses of the particles, the decay constant of the pion, and the lifetime of the $\Lambda_b$ baryon are taken from \cite{[2]}. The values of the effective Wilson coefficients are taken from \cite{[14]}. The values of the Cabibbo-Kobayashi-Maskawa (CKM) elements used are $|V_{ub}| = 0.036 \times 10^{-3}$, $|V_{us}| = 0.2255 \pm 0.0019$, $|V_{tb}| = 0.999$, $|V_{ts}| = (38.7 \pm 2.3) \times 10^{-3}$ \cite{[2]}. and the weak phase $\gamma = \theta^{\text{Phys.}}$ \cite{[17]}.

To evaluate the branching ratio for $\Lambda_b \rightarrow \Lambda \pi$ decay we need to specify the form factors describing $\Lambda_b \rightarrow \Lambda$ transition. In this analysis we use the values of the factors from \cite{[15]} which are evaluated using the light-cone sum rules. In this approach, the dependence of form factors on the momentum transfer can be parametrized as

$$\xi_i(q^2) = \frac{\xi_i(0)}{1 - a_i(q^2/m_{\Lambda_b}^2) + a_2(q^4/m_{\Lambda_b}^2)},$$ (9)

where $\xi$ denotes the form factors $g_1$ and $g_2$. The values of the parameters $\xi_i(0), a_1$, and $a_2$ have been presented in Table I. The other form factors can be related to these two as

$$g_1 = G_1, \quad g_2 = G_2 = g_3 = G_3.$$ (10)

### Table I. Numerical values of the form factors $g_1$ and $g_2$ and the parameters $a_1$ and $a_2$ involved in the double fit \cite{[9]}

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Twist 3</th>
<th>Up to twist 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_1(0)$</td>
<td>0.14$^{+0.02}_{-0.01}$</td>
<td>0.15$^{+0.02}_{-0.02}$</td>
</tr>
<tr>
<td>$a_1$</td>
<td>2.91$^{+0.10}_{-0.07}$</td>
<td>2.94$^{+0.11}_{-0.06}$</td>
</tr>
<tr>
<td>$a_2$</td>
<td>2.26$^{+0.13}_{-0.08}$</td>
<td>2.31$^{+0.14}_{-0.10}$</td>
</tr>
<tr>
<td>$g_2(0)$(10$^{-2}$ GeV$^{-1}$)</td>
<td>$-0.47^{+0.06}_{-0.06}$</td>
<td>1.3$^{+0.02}_{-0.04}$</td>
</tr>
<tr>
<td>$a_1$</td>
<td>3.40$^{+0.06}_{-0.05}$</td>
<td>2.91$^{+0.12}_{-0.09}$</td>
</tr>
<tr>
<td>$a_2$</td>
<td>2.98$^{+0.09}_{-0.08}$</td>
<td>2.24$^{+0.17}_{-0.13}$</td>
</tr>
</tbody>
</table>
FOURTH GENERATION EFFECT ON $\Lambda_b$ DECAYS

Thus, we obtain the branching ratio for the $\Lambda_b \rightarrow \Lambda \pi$ mode in the SM as

$$
\text{Br}(\Lambda_b \rightarrow \Lambda \pi) = (6.4 \pm 2.0) \times 10^{-8} \quad \text{(twist 3)},
$$

$$
\text{Br}(\Lambda_b \rightarrow \Lambda \pi) = (7.4 \pm 2.3) \times 10^{-8} \quad \text{(up to twist 6)},
$$

where we have assumed 50% uncertainties due to non-factorizable contributions. It should be noted that these values are beyond the reach of the currently running experiments and hence, observation of this mode will be a clear signal of new physics.

In the presence of a fourth generation of quarks, there will be an additional contribution due to the $t'$ quark in the electroweak penguin loops. Furthermore, it should be noted that due to the presence of the $t'$ quark the unitarity condition becomes $\lambda_u + \lambda_c + \lambda_t + \lambda_{t'} = 0$, where $\lambda_q = V_{qb}V_{qs}$.

Thus, in the presence of the fourth generation of quarks the amplitude for $\Lambda_b \rightarrow \Lambda \pi$ will become

$$
\mathcal{A}(\Lambda_b \rightarrow \Lambda \pi^0) = \sum_i \lambda_i \alpha_i \gamma_i (X + Y) u_{\Lambda_b}(p),
$$

$$
\times \bar{u}_\Lambda(p') \left[ (m_{\Lambda_b} - m_\Lambda) \lambda_i \alpha_i + g_3(q^2) \bar{m}_\pi \right],
\times \bar{u}_\Lambda(p') \left[ (m_{\Lambda_b} - m_\Lambda) \lambda_i \alpha_i + g_3(q^2) \bar{m}_\pi \right],
$$

where $X$ and $Y$ are given as

$$
X = \frac{G_F}{2} f_\pi g_1(q^2)(m_{\Lambda_b} - m_\Lambda) + g_3(q^2)\bar{m}_\pi,
$$

$$
Y = \frac{G_F}{2} f_\pi \left[ G_1(q^2)(m_{\Lambda_b} - m_\Lambda) - G_3(q^2)\bar{m}_\pi \right].
$$

The above amplitude can be represented in a more general way

$$
\mathcal{A}(\Lambda_b(p') \rightarrow \Lambda(p)\pi^0(q)) = \left[ \lambda_i \alpha_i \gamma_i \right] \left( m_{\Lambda_b} - m_\Lambda \right) \alpha_i + g_3(q^2)\bar{m}_\pi
$$

$$
\times \bar{u}_\Lambda(p') \left[ (m_{\Lambda_b} - m_\Lambda) \lambda_i \alpha_i + g_3(q^2)\bar{m}_\pi \right],
$$

where the parameters $a, b, r, s$ and $r'$ and the strong phases $\delta$ and $\delta'$ are defined as

$$
\begin{align*}
a &= |\lambda_i/\lambda_a|, & b &= |\lambda_i'/\lambda_a|, \\
\frac{r}{r'} &= \frac{a_0 - a_{17}}{a_2}, \quad \frac{s}{s'} &= \frac{a_0 - a_{17}'}{a_2}, \\
\delta &= \arg \left( \frac{a_0 - a_{17}}{a_2} \right), \quad \delta' &= \arg \left( \frac{a_0 - a_{17}'}{a_2} \right).
\end{align*}
$$

The weak phases of the CKM elements are used as follows: $\gamma$ is the phase of $V_{ub}$, $\pi$ is the phase of $V_{ts}$, and $\phi_s$ is the phase of $\lambda_{t'}$. The decay width for this process can be given by

$$
\Gamma = \frac{P_{\text{cm}}}{8\pi} \left| \lambda_i \alpha_i \right|^2 \left( \frac{m_{\Lambda_b} + m_\Lambda}{m_{\Lambda_b}^2} \right)^2 |X|^2
$$

$$
+ \left( \frac{m_{\Lambda_b} - m_\Lambda}{m_{\Lambda_b}^2} \right)^2 |Y|^2 \left[ 1 + a^2 r^2 + b^2 s^2 + 2ar \cos(\delta + \gamma) - 2br' \cos(\phi_s + \gamma + \delta') \right]
$$

$$
- 2abr' \cos(\phi_s + \delta' - \gamma)].
$$

For numerical evaluation of the branching ratio we need to know the values of the new parameters of this model. We use the allowed range for the new CKM elements as $|\lambda_i| = (0.08 \rightarrow 1.4) \times 10^{-2}$ and $\phi_s = (0 \rightarrow 80)^\circ$ for $m_{t'} = 400$ GeV, extracted using the available observables which are mediated through $b \rightarrow s$ transitions [11]. To find out the values of the QCD parameters $a_0$ and $a_{17}$, we need to evaluate the new Wilson coefficients $C_{t-10}$ due to the virtual $t'$ quark exchange in the loop. The values of these coefficients at the $M_W$ scale can be obtained from the corresponding contribution due to $t$-quark exchange by replacing the mass of the $t$ quark in the Inami-Lim functions [18] by $m_{t'}$. These values can then be evolved to the $m_t$ scale using the renormalization group equation as discussed in [19]. The values of these coefficients for a representative $t'$ mass $m_{t'} = 400$ GeV are listed in Table II.

With these inputs the variation of the branching ratio for the $\Lambda_b \rightarrow \Lambda \pi$ with $|\lambda_i'|$ is shown in Fig. 1. From the figure it can be seen that the branching ratio is significantly enhanced from its corresponding SM value and it could be easily accessible in the currently running LHCb experiment.

Now we will discuss the $\Lambda_b$ decay mode $\Lambda_b \rightarrow pK^-$, mediated through $b \rightarrow s$ transition. In the SM, it receives contributions from the color allowed tree, QCD as well as electroweak penguins. Its amplitude in the SM is given as [16]

$$
\mathcal{A}(\Lambda_b \rightarrow p\bar{K}^-) = \frac{G_F}{\sqrt{2}} f_{\bar{K}} \bar{u}_\Lambda(p') \left( \lambda_i a_1 - \lambda_i a_4 + a_{10} \
\quad + (a_5 + a_8)R_1(g_1(m_{\bar{K}}^2)(m_{\Lambda_b} - m_\Lambda) \
\quad + g_3(m_{\bar{K}}^2)m_{\bar{K}}^2 + \lambda_i a_1 - \lambda_i a_4 + a_{10} \
\quad - (a_5 + a_8)R_2(g_1(m_{\bar{K}}^2)(m_{\Lambda_b} - m_\Lambda) \
\quad - G_3(m_{\bar{K}}^2)m_{\bar{K}}^2)\gamma_5 \right| u_{\Lambda_b}(p).
$$

<table>
<thead>
<tr>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5'$</th>
<th>$C_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.06 \times 10^{-2}$</td>
<td>$-3.85 \times 10^{-2}$</td>
<td>$1.02 \times 10^{-2}$</td>
<td>$-4.43 \times 10^{-2}$</td>
</tr>
<tr>
<td>$C_{t'}$</td>
<td>$C_8'$</td>
<td>$C_9'$</td>
<td>$C_{10}$</td>
</tr>
<tr>
<td>$4.453 \times 10^{-3}$</td>
<td>$2.115 \times 10^{-3}$</td>
<td>$-0.029$</td>
<td>$0.006$</td>
</tr>
</tbody>
</table>
where

$$
R_1 = \frac{2m^2_K}{(m_b - m_\rho)(m_\rho + m_\pi)},
$$
$$
R_2 = \frac{2m^2_K}{(m_\rho + m_\pi)(m_\rho + m_\pi)}.
$$

From the above amplitude one can obtain the branching ratio using Eq. (8). Using the input parameters as discussed earlier in this section and assuming 50% uncertainties due to nonfactorizable contributions, we obtain the branching ratio in the SM

$$
\text{Br}(\Lambda_b \to pK^-) = 3.5 \times 10^{-6},
$$

which is lower than the present experimental value

$$
\text{Br}(\Lambda_b \to pK^-) = (5.6 \pm 0.8 \pm 1.5) \times 10^{-6} \ [20].
$$

Here we have used the form factors for $\Lambda_b \to p$ transitions from [21], which are evaluated in the light-front quark model. The $q^2$ dependence of the form factors is given by the following three parameters fit as

$$
\xi_i(q^2) = \frac{\xi_i(0)}{(1 - q^2/m^2_{\Lambda_i})(1 - a_1(q^2/m^2_{\Lambda_i}) + a_2(q^2/m^2_{\Lambda_i})^2),}
$$

where the values of the different fit parameters are listed in Table III.

As discussed earlier in the presence of a fourth generation of quarks the amplitude (17) will receive additional contributions due to the heavy $t'$ quark in the loop. The modified amplitude becomes

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
$\xi$ & $\xi(0)$ & $a$ & $b$ \\
\hline
$g_1$ & 0.1131 & 1.70 & 1.60 \\
$g_3$ & 0.0356 & 2.5 & 2.57 \\
$G_1$ & 0.1112 & 1.65 & 1.60 \\
$G_3$ & 0.0097 & 2.8 & 2.7 \\
\hline
\end{tabular}
\caption{Numerical values of the form factors $g_1$ and $g_2$ and the parameters $a_1$ and $a_2$ for $\Lambda_b \to p$ transition (20).}
\end{table}

\[ A(\Lambda_b \to pK^-) = i\frac{G_F}{\sqrt{2}} f_K a_\mu \left[ (\lambda_\mu a_1 - \lambda_1 a_4 + a_{10} + (a_6 + a_3)R_3) - \lambda_1 a_4 + a_{10} + (a'_6 + a'_3)R_3 \right] (m^2_K)(m_{\Lambda_i} - m_\Lambda) + g_3(m^2_K)m^2_\Lambda + \lambda_\mu a_1 - \lambda_1 a_4 + a_{10} - (a_6 + a_3)R_2 - \lambda_1(a'_4 + a'_{10}) - (a'_6 + a'_3)R_2)(G_1(m^2_K)(m_{\Lambda_i} + m_\Lambda) - G_3(m^2_K)m^2_\Lambda)\gamma_S \mu_{\Lambda_i}. \]  

Now using the values of the new Wilson coefficients $C_{3-10}$ from Table II and varying the new CKM elements between $0.0008 \leq |\lambda_i| \leq 0.014$ and $(0 \leq \phi_2 \leq 80)^\circ$, we present in Fig. 2 the variation of $\text{Br}(\Lambda_b \to pK^-)$ with $|\lambda_i|$. From the figure it can be seen that the measured branching ratio can be easily accommodated in this model.

\section{III. $\Lambda_b \to \Lambda \gamma$ Decay Width}

In this section we will consider the rare radiative decay $\Lambda_b \to \Lambda \gamma$ which is induced by the quark level transition $b \to s \gamma$. The effective Hamiltonian describing $\Lambda_b \to \Lambda \gamma$ is given as

$$
H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} \lambda_i C_7(m_\Lambda) O_7,
$$

where $C_7$ is the Wilson coefficient and $O_7$ is the electromagnetic dipole operator given as

$$
O_7 = \frac{e}{32\pi^2} F_{\mu\nu}[m_\Lambda \bar{s}\sigma^{\mu\nu}(1 + \gamma_5)b + m_\Lambda \bar{s}\sigma^{\mu\nu}(1 - \gamma_5)b].
$$

The expression for calculating the Wilson coefficient $C_7(\mu)$ is given in [22]. The matrix elements of the various hadronic currents between initial $\Lambda_b$ and the final $\Lambda$ baryon, which are parametrized in terms of various form factors as
FOURTH GENERATION EFFECT ON $\Lambda_b$ DECAYS

\[
\langle \Lambda | \bar{s} \gamma_{\mu} q^\nu b | \Lambda_b \rangle = \bar{u}_\Lambda \{ f_1 \gamma_{\mu} + i f_2 \gamma_{\mu} q^\nu + f_3 q^\nu \} u_{\Lambda_b},
\]

\[
\langle \Lambda | \bar{s} \gamma_{\mu} \gamma_5 q^\nu b | \Lambda_b \rangle = \bar{u}_\Lambda \{ f_1 \gamma_{\mu} \gamma_5 + i f_2 \gamma_{\mu} \gamma_5 q^\nu + F_3 \gamma_5 q^\nu \} u_{\Lambda_b}.
\]  

(24)

These form factors are related to the previously defined $g_1$ and $g_2$ through [15]

\[
F_1(q^2) = f_1(q^2) = q^2 g_2(q^2) = q^2 G_2(q^2),
\]

\[
F_2(q^2) = f_2(q^2) = g_1(q^2) = G_1(q^2).
\]  

(25)

Thus, one can obtain the decay width of $\Lambda_b \to \Lambda \gamma$ in the SM as

\[
\Gamma(\Lambda_b \to \Lambda \gamma) = \frac{\alpha G_F^2 |V_{tb}V_{ts}^*|^2 C_7^2(1 - x^2)^3}{32 m_{\Lambda_b}^3 \pi^3} \times (m_H^2 + m_\gamma^2 |f_2(0)|^2),
\]  

(26)

where $x = m_\Lambda/m_{\Lambda_b}$. Using the input parameters as discussed in Sec. II we obtain the branching ratio in the SM as

\[
Br(\Lambda_b \to \Lambda \gamma) = (7.93 \pm 2.31) \times 10^{-6},
\]  

(27)

which is well below the present experimental upper limit $Br(\Lambda_b \to \Lambda \gamma) < 1.3 \times 10^{-3}$ [2]. Now we would like to see the effect of the fourth quark generation on the branching ratio of $\Lambda_b \to \Lambda \gamma$. In the presence of the fourth quark generation of quarks, the Wilson coefficient $C_7$ will be modified due to the $t'$ contribution in the loop. Thus the modified parameter can be given as

\[
C_7^{t'}(\mu) = C_7(\mu) + \frac{V_{tb}^V V_{ts}^{V*}}{V_{tb}^V V_{ts}} C_7(\mu),
\]  

(28)

where $C_7^{t'}$ can be obtained from the expression of $C_7$ by replacing the mass of the $t$ quark by $m_{t'}$. The value of $C_7^{t'}$ for $m_{t'} = 400$ GeV is found to be $C_7^{t'} = -0.375$.

Thus, in SM4 the branching ratio can be given by Eq. (25) by replacing $C_7$ by $C_7^{t'}$. Now varying $\lambda_7$ between 0.0008 $\leq |\lambda_7| \leq 0.0014$ and $\phi_5$ between (0°–80°) we show in Fig. 3 the corresponding branching ratio, where we have included 30% uncertainties due to hadronic form factors. From the figure it can be seen that the branching ratio in SM4 has been significantly enhanced from its SM value and it could be easily accessible in the currently running experiments.

IV. $\Lambda_b \to \Lambda l^+ l^-$ DECAYS

The decay process $\Lambda_b \to \Lambda l^+ l^-$ is described by the quark level transition $b \to s l^+ l^-$. These processes are extensively studied in the literature [23] in various beyond the standard model scenarios. The effective Hamiltonian describing these processes can be given as [19]

\[
\mathcal{H}_{\text{eff}} = \frac{G_F \alpha}{\sqrt{2} \pi} V_{tb} V_{ts}^* \left[ C_9^{\text{eff}} (\bar{s} \gamma_\mu Lb)(\bar{l} \gamma^\mu l) + C_{10} (\bar{s} \gamma_\mu Lb) \times (\bar{l} \gamma^\mu l) - 2C_7^{\text{eff}} m_b (\bar{s} \bar{d} \gamma^\mu q^\nu \frac{q^\nu}{q^2} Rb)(\bar{l} \gamma^\mu l) \right].
\]

(29)

where $q$ is the momentum transferred to the lepton pair, given as $q = p_- + p_+$, where $p_-$ and $p_+$ are the momenta of the leptons $l^-$ and $l^+$, respectively. $L, R = (1 \pm \gamma_5)/2$ and $C_i$'s are the Wilson coefficients evaluated at the $b$ quark mass scale. The values of these coefficients in next-leading-logarithmic (NLL) order are $C_9^{\text{eff}} = -0.31$, $C_9 = 4.154$, and $C_{10} = -4.261$ [24].

The coefficient $C_9^{\text{eff}}$ has a perturbative part and a resonance part which comes from the long distance effects due to the conversion of the real $c\bar{c}$ into the lepton pair $l^+ l^-$. Therefore, one can write it as

\[
C_9^{\text{eff}} = C_9 + Y(s) + C_9^{\text{res}},
\]

(30)

where $s = q^2$ and the function $Y(s)$ denotes the perturbative part coming from one-loop matrix elements of the four quark operators and is given by [19]

\[
Y(s) = g(m_c, s)(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6)
- \frac{1}{2} g(0, s)(C_3 + 3C_4) - \frac{1}{2} g(m_b, s)(4C_3 + 4C_4
+ 3C_5 + C_6) + \frac{3}{2}(3C_3 + C_4 + 3C_5 + C_6),
\]

(31)

where

\[
g(m_b, s) = -\frac{8}{9} \ln(m_b^2/m_b^{\text{pole}}) + \frac{8}{27} + \frac{4}{9} y_i - \frac{2}{9} (2 + y_i)
\times \sqrt{[1 - y_i]} \left[ \Theta(1 - y_i) \left[ \ln \left( \frac{1 + \sqrt{1 - y_i}}{1 - \sqrt{1 - y_i}} \right) - i \pi \right]
\right.
\]

\[
+ \Theta(y_i - 1) 2 \arctan \frac{1}{\sqrt{y_i - 1}},
\]

(32)

with $y_i = 4m_i^2/s$. The values of the coefficients $C_i$'s in NLL order are taken from [24].
The long distance resonance effect is given as [25]
\[
C_{\psi}^{\psi} \approx \frac{3\pi}{\alpha^2} (3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) \\
\times \sum_{\nu_1=\phi(153), \phi(65)} \kappa_{\nu_1} \frac{\Gamma(V_1 \to l^+ l^-)}{m_{\nu_1}^2 - s - im_{\nu_1} \Gamma_{V_1}}.
\]
(33)

The phenomenological parameter \( \kappa \) is taken to be 2.3, so as to reproduce the correct branching ratio of \( \text{Br}(B \to J'/\psi K^{*+} l^- l^+) = \text{Br}(B \to J'/\psi K^*)\text{Br}(J'/\psi \to l^- l^+) \).

The matrix elements of the various hadronic currents in (29) between initial \( \Lambda_b \) and the final \( \Lambda \) baryon are parameterized in terms of various form factors as defined in Eqs. (3) and (24). Thus, using these matrix elements, the transition amplitude can be written as
\[
\mathcal{M}(\Lambda_b \to \Lambda l^+ l^-) = \frac{G_F \alpha}{\sqrt{2} \pi} V_{tb} V_{ts}^* \left[ \bar{u}_\Lambda (\gamma^\mu (A_1 P_R + B_1 P_L) + i\sigma^{\nu\rho} q_\nu (A_2 P_R + B_2 P_L) u_\Lambda) \right. \\
+ \bar{\nu}_\mu (\gamma^\mu (D_1 P_R + E_1 P_L) + i\sigma^{\nu\rho} q_\nu (D_2 P_R + E_2 P_L) + q^\mu (D_3 P_R + E_3 P_L) u_\Lambda),
\]
(34)
where the various parameters \( A_i, B_i \) and \( D_j, E_j \) \( (i = 1, 2 \) and \( j = 1, 2, 3 \) ) are defined as

\[
\mathcal{K}_0(s) = 32 m_\Lambda^2 m_\Lambda^2 \delta (1 + r - \hat{s})(|D_1|^2 + |E_1|^2) + 64 m_\Lambda^3 (1 - r - \hat{s}) \Re(D_1^* E_3 + D_3^* E_1^*) \\
+ 64 m_\Lambda^3 \sqrt{6 m_\Lambda^2 - \delta^3}(2 m_\Lambda \delta \Re(D_1^* E_3) + (1 - r + \hat{s}) \Re(D_1^* D_3 + E_1^* E_3)) \\
+ 32 m_\Lambda^3 (2 m_\Lambda^2 + m_\Lambda^3 \delta)(1 - r + \hat{s}) m_\Lambda \sqrt{2} \Re(A_1^* A_2 + B_1^* B_2 - m_\Lambda^2 (1 - r - \hat{s}) \Re(A_1^* B_2 + B_1^* A_2)) \\
- 2\sqrt{2} \Re(A_1^* B_1 + m_\Lambda^2 \delta \Re(A_1^* B_2)) + 8 m_\Lambda^3 (4 m_\Lambda^2(1 + r - \hat{s}) + m_\Lambda^3 [(1 - r)^2 - \delta^2]) |A_1|^2 + |B_1|^2 \\
+ 8 m_\Lambda^3 (4 m_\Lambda^2 (\lambda + (1 + r - \hat{s}) \delta) + m_\Lambda^3 [(1 - r)^2 - \delta^2]) |A_2|^2 + |B_2|^2 \\
- m_\Lambda^3 [(1 - r)^2 - \delta^2] |D_1|^2 + |E_1|^2) + 8 m_\Lambda^3 \delta \nu_1 (-8 m_\Lambda^2 \delta \sqrt{2} \Re(D_1^* E_2) + 4(1 - r + \hat{s}) \\
\times \sqrt{2} \Re(D_1^* D_2 + E_1^* E_2) - 4(1 - r - \hat{s}) \Re(D_1^* E_2 + D_2^* E_1) + m_\Lambda^3 [(1 - r)^2 - \delta^2] |D_2|^2 + |E_2|^2),
\]
(38)

\[
\mathcal{K}_1(s) = -16 m_\Lambda^3 \delta \nu_1 \sqrt{\lambda} (2 \Re(A_1^* D_1) - 2 \Re(B_1^* E_1) \\
+ 2 m_\Lambda \Re(B_1^* D_2 - B_2^* D_1 + A_1^* E_1 - A_1^* E_2)) \\
+ 32 m_\Lambda^3 \delta \nu_1 \sqrt{\lambda} (m_\Lambda (1 - r) \Re(A_1^* D_2 - B_2^* E_1) \\
+ \sqrt{2} \Re(A_2^* D_1 + A_1^* D_2 - B_2^* E_1 - B_1^* E_2)),
\]
(39)

and
\[
\mathcal{K}_2(s) = 8 m_\Lambda^3 \nu_1^2 \lambda \delta ((|A_2|^2 + |B_2|^2 + |D_2|^2 + |E_2|^2) \\
- 8 m_\Lambda^3 \nu_1^2 \lambda (|A_1|^2 + |B_1|^2 + |D_1|^2 + |E_1|^2)).
\]
(40)

The dilepton mass spectrum can be obtained from (36) by integrating out the angular dependent parameter \( z \) which yields
\[
\left( \frac{d\Gamma}{ds} \right)_0 = \frac{G_F^2 \alpha^2}{2^{13} \pi^5 m_\Lambda} |V_{tb} V_{ts}^*|^2 \nu_1 \sqrt{\lambda} \left[ \mathcal{K}_0(s) + \frac{1}{3} \mathcal{K}_2(s) \right],
\]
(41)

where \( \lambda \) is the shorthand notation for \( \lambda(1, r, \delta) \). The limits for \( s \) are
\[
4 m_\Lambda^2 \leq s \leq (m_{\Lambda_b} - m_\Lambda)^2.
\]
(42)

Apart from the branching ratio in semileptonic decay, there are also other observables which are sensitive to new physics contributions in \( b \to s \) transition. One such
The weak mixing angle is a very powerful tool for looking for new physics. The normalized forward-backward asymmetry is obtained by integrating the double differential decay width \(d^2\Gamma/d\bar{s}dz\) with respect to the angular variable \(z\)

\[
A_{FB}(s) = \frac{\int_0^s \frac{d^2\Gamma}{d\bar{s}dz} dz - \int_{s}^0 \frac{d^2\Gamma}{d\bar{s}dz} dz}{\int_0^s \frac{d^2\Gamma}{d\bar{s}dz} dz + \int_{s}^0 \frac{d^2\Gamma}{d\bar{s}dz} dz}.
\]

Thus one obtains from (36)

\[
A_{FB}(s) = \frac{\mathcal{K}_1(s)}{\mathcal{K}_0(s) + \mathcal{K}_2(s)/3}. \tag{44}
\]

The FB asymmetry becomes zero for a particular value of dilepton invariant mass. Within the SM, the zero of \(A_{FB}(s)\) appears in the low \(q^2\) region, sufficiently away from the charm resonance region and hence can be predicted precisely. The position of the zero value of \(A_{FB}\) is very sensitive to the presence of new physics.

For numerical evaluation we use the input parameters as presented in the previous sections. The quark masses (in GeV) used are \(m_b = 4.6, m_c = 1.5\), and \(\alpha = 1/128\) and the weak mixing angle \(\sin^2\theta_W = 0.23\). The variation of differential branching ratios (41) and the forward-backward asymmetries (44) for the processes \(\Lambda_b \rightarrow \Lambda \mu^+ \mu^-\) and \(\Lambda_b \rightarrow \Lambda \tau^+ \tau^-\) in the standard model are shown in Figs. 4 and 5, respectively.

As discussed earlier in the presence of the fourth generation, the Wilson coefficients \(C_{7,9,10}\) will be modified due to the new contributions arising from the virtual \(t'\) quark in the loop. Thus, these coefficients will be modified as

\[
C_7^{t'}(\mu) = C_7(\mu) + \frac{\lambda_{t'}}{\lambda_{t}} C_7(\mu),
\]

\[
C_9^{t'}(\mu) = C_9(\mu) + \frac{\lambda_{t'}}{\lambda_{t}} C_9(\mu),
\]

\[
C_{10}^{t'}(\mu) = C_{10}(\mu) + \frac{\lambda_{t'}}{\lambda_{t}} C_{10}(\mu).
\]

The new coefficients \(C_{7,9,10}^{t'}\) can be calculated at the \(M_W\) scale by replacing the \(t\)-quark mass by \(m_t'\) in the loop functions. These coefficients are then to be evolved to the \(b\) scale using the renormalization group equation as discussed in [19]. The values of the new Wilson coefficients at the \(m_b\) scale for \(m_{t'} = 400\) GeV are given by \(C_7(m_b) = -0.355, C_9(m_b) = 5.831,\) and \(C_{10} = -17.358\).

Thus, one can obtain the differential branching ratio and the forward-backward asymmetry in SM4 by replacing \(C_{7,9,10}\) in Eqs. (41) and (44) by \(C_{7,9,10}^{t'}\). Using the values of the \(|\lambda_{t'}|\) and \(\phi_{t'}\) for \(m_{t'} = 400\) GeV, differential branching ratio and the forward-backward asymmetry for

\[\begin{align*}
\text{FIG. 4 (color online).} & \quad \text{The differential branching ratio } d\text{Br}/ds \text{ versus } s \text{ (left panel) and the forward-backward asymmetry } A_{FB}(s) \text{ versus } s \text{ (right panel) for the process } \Lambda_b \rightarrow \Lambda \mu^+ \mu^-.
\end{align*}\]

\[\begin{align*}
\text{FIG. 5 (color online).} & \quad \text{Same as Fig. 4 for the process } \Lambda_b \rightarrow \Lambda \tau^+ \tau^-.
\end{align*}\]
\( \Lambda_b \rightarrow \Lambda \mu^+ \mu^- \) are presented in Fig. 6, where we have not considered the contributions from intermediate charmonium resonances. From the figure it can be seen that the differential branching ratio of this mode is significantly enhanced from its corresponding SM value, whereas the forward-backward asymmetry is slightly reduced with respect to its SM value. However, the zero position of the FB asymmetry remains unchanged in the fourth quark generation model. Similarly for the process \( \Lambda_b \rightarrow \Lambda \tau^+ \tau^- \) as seen from Fig. 7, the branching ratio is significantly enhanced from its SM value, whereas the FB asymmetry remains almost unaffected in the SM4.

We now proceed to calculate the total decay rates for \( \Lambda_b \rightarrow \Lambda l^+ l^- \) for which it is necessary to eliminate the backgrounds coming from the resonance regions. This can be done by using the following veto windows so that the backgrounds coming from the dominant resonances \( \Lambda_b \rightarrow \Lambda J/\psi (\psi') \) with \( J/\psi (\psi') \rightarrow l^+ l^- \) can be eliminated,

- \( \Lambda_b \rightarrow \Lambda \mu^+ \mu^- : m_{\mu^+ \mu^-} \rightarrow m_{\mu^+ \mu^-} < m_{J/\psi} + 0.02, \)
- \( \Lambda_b \rightarrow \Lambda \psi' \rightarrow m_{\psi'} \rightarrow m_{\mu^+ \mu^-} < m_{\psi'} + 0.02, \)
- \( \Lambda_b \rightarrow \Lambda \tau^+ \tau^- : m_{\tau^+ \tau^-} < m_{\psi'} + 0.02. \)

Using these veto windows we obtain the branching ratios for semileptonic rare \( \Lambda_b \) decays which are presented in Table IV. It is seen from the table that the branching ratios obtained in the fourth quark generation model are reasonably enhanced from the corresponding SM values and could be observed in the LHCb experiment.

### V. CONCLUSION

In this paper we have studied several rare decays of \( \Lambda_b \) baryon, i.e., \( \Lambda_b \rightarrow \Lambda \pi, \Lambda_b \rightarrow p K^-, \Lambda_b \rightarrow \Lambda \gamma, \) and \( \Lambda_b \rightarrow \Lambda l^+ l^- \) in the fourth quark generation model. This model is a very simple extension of the standard model with three generations and it provides a simple explanation for several indications of new physics that have been observed.
observed involving $CP$ asymmetries in the $B, B_s$ decays for $m_l^*$ in the range of (400–600) GeV. We found that in this model the branching ratios of the various decay modes considered here ($\Lambda_b \rightarrow \Lambda \pi, \Lambda_b \rightarrow pK^-, \Lambda_b \rightarrow \Lambda\gamma$, and $\Lambda_b \rightarrow \Lambda l^+l^-$) are significantly enhanced from their corresponding SM values. However the forward-backward asymmetries in the $\Lambda_b \rightarrow \Lambda l^+l^-$ processes do not differ much from those of the SM expectations. The zero point of the $F_{AB}$ for $\Lambda_b \rightarrow Ll^+l^-$ process is also found to be unaffected in this model.

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