Determination of possible mass Range of Dark matter in the Inert Scalar Triplet Model

Chayan Majumdar

A Thesis Submitted to
Indian Institute of Technology Hyderabad
In Partial Fulfillment of the Requirements for
The Degree of Master of Science

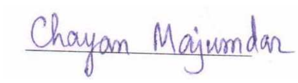
भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

Department of Physics

April 29, 2015

Declaration

I declare that this written submission represents my ideas in my own words, and where ideas or words of others have been included, I have adequately cited and referenced the original sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented or fabricated or falsified any idea/data/fact/source in my submission. I understand that any violation of the above will be a cause for disciplinary action by the Institute and can also evoke penal action from the sources that have thus not been properly cited, or from whom proper permission has not been taken when needed.



(Signature)

Chayan Majumdar

(Chayan Majumdar)

PH13M1005

(Roll No.)

Approval Sheet

This thesis entitled "**Determination of Possible Mass Range of Dark Matter in the Inert Scalar Triplet Model**" by Chayan Majumdar is approved for the degree of Master of Science from IIT Hyderabad.

Dr. Prem Pal
Coordinator/Examiner

Dr. Narendra Sahu
Advisor/Examiner

18.5.15
Dr. J.Suryanarayana
Examiner

Dr. J.Mohanty
Examiner

05/05/15
Dr. Vandana Sharma
Examiner

Dr. S.Hundi
Examiner

Acknowledgements

At first,I want to convey my acknowledgement to my guide as well as supervisor,**Dr.Narendra Sahu Sir (**`nsahu@iith.ac.in`
for his thankful consideration as a project fellow under his supervision as well as his great co-operation in any type of need to complete this project successfully.then,I also want to acknowledge to Supriya Senapati(ph13m1015@iith.ac.in)
for her help as a project partner as well as Parswa Nath(ph13m1009@iith.ac.in) and Animesh Adhikari(ph13m1003@iith.ac.in)
for their help to develop the Latex knowledge of myself as well as to guide me in the computer programming(C++,Gnuplot).

Dedication

My project work is dedicated to my beloved parents, I am here today for their blessing and inspiration.

Abstract

Dark matter is a motivation to search for physics beyond the Standard Model.In this project-report,We discuss briefly about the Inert Scalar Triplet Model(ITM) to find out the possibility of stable Dark Matter candidate stabilized by Z_2 extension of the standard Model and the constraints of the constants in the Lagrangian.Here we can establish the relationship between **relic density of the dark matter in the present day universe and the mass of the expected dark matter candidate** from the scattering cross-section calculation.It gives rise to the plot between the two quantities mentioned above and from there we can easily determine the possible mass range according to the relic density range of the dark matter.

Contents

Chapter 1

The Standard Model

1.1 Introduction

We present here a primer on the Standard Model of the electroweak interaction.The pioneer attempt to incorporate the V-A structure in a gauge theory for the weak interactions was made by **Bludman** in 1958.This model, based on the SU(2) weak isospin group, also required three vector bosons.American Theoretical physicist **Glashow** in 1961 noticed that in order to accommodate both weak and electromagnetic interactions we should go beyond the SU(2) isospin structure.He suggested that the gauge group *SU(2)*$\otimes$*U(1)*, where the U(1) was associated to the leptonic hypercharge (Y) that is related to the weak isospin (T) and the electric charge through the analogous of the **"Gell-Mann-Nishijima formula"** ($Q = T_3 + Y/2$). The theory now requires four gauge bosons: a triplet (W^1, W^2, W^3) associated to the generators of SU(2) and a neutral field (B) related to U(1).Here The charged weak bosons appear as a linear combination of W^1 and W^2, while the massless photon and a neutral weak boson Z^0 are both given by a mixture of W^3 and B.In 1967,**Weinberg** and independently **Salam** in 1968, employed the idea of spontaneous symmetry breaking and the Higgs mechanism to give mass to the weak bosons and, at the same time, to preserve the gauge invariance in this theory.The **Glashow–Weinberg–Salam model** is known, at the present time, as the **Standard Model of Electroweak Interactions**, reflecting its impressive success.

1.2 Right and Left Handed Fermions

Before the introduction of the Standard Model, let us make an interlude and discussion some properties of the fermionic helicity states. At high energies (i.e. for $E \gg m$), the Dirac spinors

$$\text{u(p,s) and } v(p,s) = C\overline{u}^T(p,s) = i\gamma_2 u^*(p,s)$$

are the eigenstates of the γ_5 matrix.

It is convenient to define the helicity projectors:

$$L = \tfrac{1}{2}(1-\gamma_5) \text{ and } R = \tfrac{1}{2}(1+\gamma_5)$$

For the conjugate spinors we have,

$$\overline{\Psi}_L = (L\Psi)^\dagger \gamma_0 = \Psi^\dagger L^\dagger \gamma_0 = \Psi^\dagger R \gamma_0 = \overline{\Psi} R$$
$$\overline{\Psi}_R = \overline{\Psi} L$$

First of all, here we notice that fermion mass term mixes right and left handed fermion components,

$$\overline{\Psi}\Psi = \overline{\Psi}_R \Psi_L + \overline{\Psi}_L \Psi_R$$

On the other hand, the electromagnetic current, does not mix those components, i.e.

$$\overline{\Psi}\gamma^\mu \Psi = \overline{\Psi}_R \gamma^\mu \Psi_R + \overline{\Psi}_L \gamma^\mu \Psi_L$$

Finally, the (V-A) fermionic weak current can be written in terms of the helicity states as,

$$\overline{\Psi}_L \gamma^\mu \Psi_L = \overline{\Psi} R \gamma^\mu L \Psi = \overline{\Psi}\gamma^\mu L^2 \Psi = \overline{\Psi}\gamma^\mu L \Psi = \tfrac{1}{2}\overline{\Psi}\gamma^\mu (1-\gamma_5)\Psi$$

what shows that only left–handed fermions play a role in weak interactions.

1.3 Choosing the Gauge Group

We see that the weak current, for a generic lepton l, is given by,

$$J_\mu^\dagger = \bar{l}\gamma_\mu (1-\gamma_5)\nu = 2\bar{l}_L \gamma_\mu \nu_L$$

If we introduce the left–handed isospin doublet ($T = 1/2$),

$$R = l_R, L = \begin{pmatrix} \nu_L \\ l_L \end{pmatrix} \tag{1.1}$$

where the $T_3 = +1/2$ and $T_3 = -1/2$ components are the left handed parts of the neutrino and of the charged leptons respectively.

The charged weak current can be written in terms of leptonic isospin currents:

$$J_\mu^i = \overline{L}\gamma_\mu \tfrac{\tau^i}{2} L$$

where τ^i are the three well known Pauli Matrices.In an explicit form we get,

$$J_\mu^1 = \tfrac{1}{2}(\bar{l}_L \gamma_\mu \nu_L + \overline{\nu}_L \gamma_\mu l_L)$$
$$J_\mu^2 = \tfrac{i}{2}(\bar{l}_L \gamma_\mu \nu_L - \overline{\nu}_L \gamma_\mu l_L)$$
$$J_\mu^3 = \tfrac{1}{2}(\overline{\nu}_L \gamma_\mu \nu_L + \bar{l}_L \gamma_\mu l_L)$$

Therefore, the weak charged current, that couples with intermediate vector boson $W_\mu^\dagger$, can be written in terms of J^1 and J^2 as,

$$J_\mu^\dagger == 2(J_\mu^1 - iJ_\mu^2)$$

Hypercharge current:

$$J_\mu^Y = -(\overline{\nu}_L\gamma_\mu\nu_L + \bar{l}_L\gamma_\mu l_L + 2\bar{l}_R\gamma_\mu l_R)$$

Electromagnetic current:

$$J_\mu^{em} = J_\mu^3 + \frac{1}{2}J_\mu^Y$$

For the Gauge Group $SU(2)_L \otimes U(1)_Y$,the next step is to introduce gauge fields corresponding to each generator, that is,

$$SU(2)_L \to W_\mu^1, W_\mu^2, W_\mu^3$$
$$U(1)_Y \to B_\mu$$

Here the Lagrangian for the Gauge field becomes,

$$L_{gauge} = -\frac{1}{4}W_{\mu\nu}^i W^{i\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}$$

For the leptons, we write the free Lagrangian,

$$L_{leptons} = \bar{l}i\eth l + \overline{\nu}i\eth\nu$$

The next step is to introduce the fermion-gauge boson coupling via the covariant derivative, i.e.

$$L \Rightarrow \partial_\mu + i\tfrac{g}{2}\tau^i W_\mu^i + i\tfrac{g\prime}{2}YB_\mu$$
$$R \Rightarrow \partial_\mu + i\tfrac{g\prime}{2}YB_\mu$$

Left handed part of the Lagrangian becomes,

$$L_{leptons}^L = -g\overline{L}\gamma^\mu(\tfrac{\tau^1}{2}W_\mu^1 + \tfrac{\tau^2}{2}W_\mu^2)L - g\overline{L}\gamma^\mu\tfrac{\tau^3}{2}LW_\mu^3 - \tfrac{g\prime}{2}Y\overline{L}\gamma^\mu LB_\mu$$

The first term is charged and can be written as

$$L_{leptons}^{L(\pm)} = -\tfrac{g}{2\sqrt{2}}[\overline{\nu}\gamma^\mu(1-\gamma_5)lW_\mu^+ + \bar{l}\gamma^\mu(1-\gamma_5)\nu W_\mu^-]$$

reproduces exactly the (V-A) structure of the weak charged current.From here we can get the definition of charged gauge bosons as,

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2)$$

From low-energy phenomenology we get that,$G_W = \dfrac{g}{2\sqrt{2}}$ and also we can conclude that $\dfrac{g}{2\sqrt{2}} = \sqrt{\frac{M_W^2 G_F}{\sqrt{2}}}$
Now let us treat the neutral piece of $L_{leptons}$ that contains both left and right fermion components,

$$L_{leptons}^{(L+R)(0)} = -gJ_3^\mu W_\mu^3 - \tfrac{g\prime}{2}J_Y^\mu B_\mu$$

In order to obtain the right combination of fields that couples to the electromagnetic current, let us make the rotation in the neutral fields, defining the new fields A and Z by,

$$W_\mu^3 = sin\theta_W A_\mu + cos\theta_W Z_\mu$$
$$B_\mu = cos\theta_W A_\mu - sin\theta_W Z_\mu$$

where θ_W is the **'Weinberg Angle'**.
So,the relation between coupling constants and the Weinberg angle are given by ,

$$sin\theta_W = \frac{g\prime}{\sqrt{g^2 + g\prime^2}}$$
$$cos\theta_W = \frac{g}{\sqrt{g^2 + g\prime^2}}$$

We easily identify the electromagnetic current coupled to the photon field A_μ and the electromagnetic charge as,

$$e = gsin\theta_W = g\prime cos\theta_W$$

The Standard Model introduces a new ingredient, weak interactions without change of charge, and make a specific prediction for the vector (V) and axial (A)-couplings of the Z to the fermions,

$$g_V^i = T_3^i - 2Q_i sin^2\theta_W \text{ and } g_A^i = T_3^i$$

Up to now we have in the theory:

•**4 massless gauge fields** i.e, W_μ^i, B_μ
•**2 massless fermions** i.e,ν,l

1.4 The Higgs Mechanism and W and Z mass

In order to apply the Higgs mechanism to give mass to $W^\pm$ and Z^0, let us introduce the scalar doublet as

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad (1.2)$$

We introduce the Lagrangian here

$$L_{scalar} = \partial_\mu \Phi^\dagger \partial^\mu \Phi - V(\Phi^\dagger \Phi) \quad (1.3)$$

where

$$V(\Phi^\dagger \Phi) = \mu^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2$$

In order to maintain the Gauge invariance we can get the new covariant derivative here,

$$\partial_\mu \rightarrow D_\mu = \partial_\mu + ig\tfrac{\tau^i}{2} W_\mu^i + i\tfrac{g\prime}{2} Y B_\mu$$

We can choose the vacuum expectation value of the Higgs field as,

$$< \Phi >_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \tag{1.4}$$

where

$$v = \sqrt{-\frac{\mu^2}{\lambda}}$$

Since we want to preserve the exact electromagnetic symmetry to maintain the electric charged conserved, we must break the original symmetry group as,

$$SU(2)_L \otimes U(1)_Y \to U(1)_{em}$$

In this case the corresponding gauge boson, the photon, will remain massless,here the operator Q annihilates the vacuum,i.e, $Q < \Phi >_0 = 0$

The other gauge bosons, corresponding to the broken generators T_1, T_2, and $(T_3 - Y/2) = 2T_3 - Q$ should acquire mass.

$$\Phi = exp(i\frac{\tau^i}{2}\frac{\chi_i}{v}) \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix} \tag{1.5}$$

or,

$$\Phi = \begin{pmatrix} iw^+ \\ \frac{v+H-iz^0}{\sqrt{2}} \end{pmatrix} \tag{1.6}$$

where $w^\pm$ and z^0 are the **Goldstone bosons**.The quadratic terms in the vector fields, are,

$$\frac{g^2v^2}{4}W^+_\mu W^{-\mu} + \frac{g^2v^2}{8cos^2\theta_W}Z^\mu Z_\mu$$

When compared with the usual mass terms for a charged and neutral vector bosons,

$$M_W^2 W^+_\mu W^{-\mu} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu$$

So we can easily identify,

$$M_W = \frac{gv}{2} \text{ and } M_Z = \frac{gv}{2cos\theta_W}$$

We can see from that no quadratic term in A_μ appears, and therefore, the photon remains massless, as we could expect since the $U(1)_{em}$ remains as a symmetry of the theory.We obtain for the vacuum expectation value $v = 246GeV$ and the mass of the W and Z bosons as $M_W \sim 80GeV$ and $M_Z \sim 90GeV$.We assumed a experimental value for $sin^2\theta_W \sim 0.22$.We can learn from that one scalar boson, out of the four degrees of freedom, is remnant of the symmetry breaking. The search for the so called **Higgs boson**, remains as one of the major challenges of the experimental high energy physics.It gives rise to terms involving exclusively the scalar field H, namely,

$$-\tfrac{1}{2}(-2\mu^2)H^2 + \tfrac{1}{4}\mu^2 v^2(\frac{4}{v^3}H^3 + \frac{H^4}{v^4} - 1)$$

We can also identify the Higgs boson mass term with $M_H = \sqrt{-2\mu^2}$

In spite of predicting the existence of the Higgs boson, the Standard Model does not give a hint on the value of its mass since μ^2 is a priori unknown.

1.5 Introducing the quarks

In order to introduce the strong interacting particles in the Standard Model we shall first examine what happens with the hadronic neutral current when the Cabibbo angle is taken into account.

$$J_\mu^H(0) = \overline{u}\gamma_\mu(1-\gamma_5)u + cos^2\theta_C\overline{d}\gamma_\mu(1-\gamma_5)d + sin^2\theta_C\overline{s}\gamma_\mu(1-\gamma_5)s + cos\theta_C sin\theta_C[\overline{d}\gamma_\mu(1-\gamma_5)s + \overline{s}\gamma_\mu(1-\gamma_5)d]$$

We should notice that the last term generates flavor changing neutral currents (FCNC), i.e. transitions like $d + \overline{s} \leftrightarrow \overline{d} + s$ In 1970, Glashow, Iliopoulos, and Maiani proposed the GIM mechanism.They consider a fourth quark flavor, the charm (c), already introduced by Bjorken and Glashow in 1963.Therefore, the charged weak couplings quark-gauge bosons, is given by,

$$L^{\pm}_{quarks} = \frac{g}{2\sqrt{2}}[\overline{u}\gamma^\mu(1-\gamma_5)d\prime + \overline{c}\gamma^\mu(1-\gamma_5)s\prime]W^+_\mu + h.c$$

The neutral current interaction of the quarks become,

$$L^{(0)}_{quarks} = -\frac{g}{2cos\theta_W}\Sigma\overline{\Psi}_q\gamma^\mu(g^q_V - g^q_A\gamma_5)\Psi_q Z_\mu$$

1.6 The Standard Model Lagrangian

1.6.1 Gauge-boson+Scalar

$$
\begin{aligned}
L_{gauge} + L_{Scalar} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
&-\frac{1}{2}W^{+}_{\mu\nu}W^{-\mu\nu}+ \\
&+M_W^2 W^{+}_{\mu}W^{-\mu} - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} \\
&+M_Z^2 Z_{\mu}Z^{\mu} + \frac{1}{2}\partial_{\mu}H\partial^{\mu}H - \frac{1}{2}M_H^2 H^2 \\
&+W^+W^-A + W^+W^-Z + W^+W^-AA + W^+W^-ZZ + W^+W^-AZ \\
&+W^+W^-W^+W^- + HHH + HHHH + W^+W^-H + W^+W^-HH + ZZH + ZZHH
\end{aligned}
$$

1.6.2 Leptons+Yukawa

$$
\begin{aligned}
L_{leptons} + L^{l}_{yuk} = &\Sigma_l \bar{l}(i\eth - m_l)l \\
&+\Sigma_{\nu_l}\overline{\nu_l}(i\eth)\nu_l + \bar{l}lA \\
&+\nu_l l W^+ + \bar{l}\nu_l W^- + \bar{l}lZ \\
&+\overline{\nu_l}\nu_l Z + \bar{l}lH
\end{aligned}
$$

1.6.3 Quarks+Yukawa

$$
\begin{aligned}
L_{quarks} + L^{q}_{yuk} = &\Sigma_q \overline{q}(i\eth - m_q)q \\
&+\overline{q}qA + \overline{u}d\prime W^+ + \overline{d\prime}uW^- \\
&+\overline{q}qZ + \overline{q}qZ
\end{aligned}
$$

1.7 Beyond Standard Model

1.7.1 Links with Physics Beyond the Standard Model

In the present day universe,according to Planck's recent paper,we can say the universe is constituted by only 4 percent of real matter.But within the remaining part,there is about 26 percent matter which accounts to dark matter which cannot be explained by Standard Model. So,it is necessary to study the physics beyond the standard model.

From recent Planck paper,we can conclude that,

1.7.2 Evidence of Dark Matter

The Galactic Scale

The most convincing and direct evidence for dark matter on galactic scales comes from the observations of the rotation curves of galaxies, namely the graph of circular velocities of stars and gas as a function of their distance

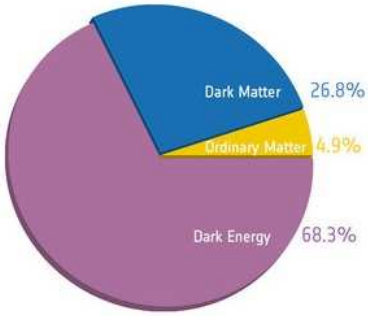


Figure 1.1: The abundance of dark matter in present universe

from the galactic center.

Rotation curves are usually obtained by combining observations of the 21cm line with optical surface photometry. Observed rotation curves usually exhibit a characteristic flat behavior at large distances, i.e. out towards, and even far beyond, the edge of the visible disks.

In Newtonian dynamics the circular velocity is expected to be,

$$v(r) = \sqrt{\frac{GM(r)}{r}}$$

where, as usual,$M(r) = 4\pi \int \rho(r) r^2 dr$ and and $\rho(r)$ is the mass density profile, and should be falling $\propto \frac{1}{\sqrt{r}}$ beyond the optical disc. The fact that v(r) is approximately constant implies the existence of an halo with M(r)$\propto$r and $\rho \propto \frac{1}{r^2}$.

Among the most interesting objects, from the point of view of the observation of rotation curves, are the so-called Low Surface Brightness (LSB) galaxies, which are probably everywhere dark matter-dominated, with the observed stellar populations making only a small contribution to rotation curves. Such a property is extremely important because it allows one to avoid the difficulties associated with the deprojection and disentanglement of the dark and visible contributions to the rotation curves.

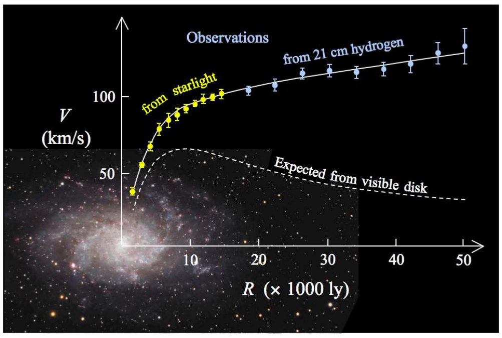


Figure 1.2: Rotation curve of the typical spiral galaxy M 33 (yellow and blue points with errorbars) and the predicted one from distribution of the visible matter (white line). The discrepancy between the two curves is accounted for by adding a dark matter halo surrounding the galaxy

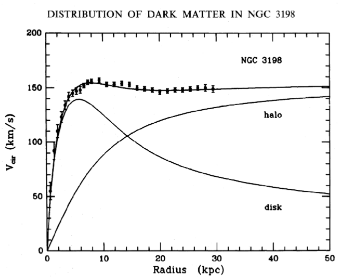


Figure 1.3: Here's the rotation curve which comes from the Doppler shift measurements of the 21.1 cm line

Chapter 2

Introduction to cosmology and Equilibrium thermodynamics in the expanding universe

2.1 Introduction

In this chapter,at first we discuss about some basic introduction to the Cosmology and then we discuss about the thermodynamics in the early universe.**Hot Big-Bang** Model or the **Friedmann-Robertson-Walker** Cosmological Model is the most discussed Model about the structure of the Present Universe.The model is becoming so successful that it is known as the "**The Standard Cosmological Model**".

2.2 Introduction to Cosmology

2.2.1 Robertson-Walker Metric

The distribution of matter and radiation in the observable universe is **homogeneous and isotropic**. The universe is spatially homogeneous and isotropic on scales as large as the Hubble Volume.This is known as the "**Cosmological Principle**". Here we can introduce the maximally symmetric Robertson-Walker metric,

$$ds^2 = dt^2 - R^2(t)\{\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2\} \tag{2.1}$$

Here k can be taken as +1,-1 and 0 for the surfaces with constant positive,negative and zero spatial curvature,respectively.The spatial part of the metric is denoted by,

$$\overrightarrow{dl}^2 = h_{ij} dx^i dx^j$$

Here,three-dimensional tensorial notations can be treated as,
Riemann tensor: $3_{R_{ijkl}} = \frac{k}{R^2(t)}(h_{ik}h_{jl} - h_{il}h_{kj})$
Ricci tensor: $3_{R_{ij}} = \frac{2k}{R^2(t)}h_{ij}$
Ricci scalar: $3_R = \frac{6k}{R^2(t)}$
Although general relativity allows one to formulate the laws of physics using arbitrary coordinates, some coordinate

choices are more natural (easier to work with). Comoving coordinates are an example of such a natural coordinate choice. They assign constant spatial coordinate values to observers who perceive the universe as isotropic. Such observers are called "**Comoving**" observers because they move along with the Hubble flow.

Particle Kinematics

$$|\overrightarrow{v_0}| = |\overrightarrow{v_1}| \frac{R(t_1)}{R(t_0)}$$

Kinematics of the RW Metric can be written as,

$$\frac{\lambda_1}{\lambda_0} = \frac{R(t_1)}{R(t_0)}$$

From the CMB radiation and the equation above we can conclude that the "**Universe is Expanding**"
Red shift: $1 + z = \dfrac{\lambda_0}{\lambda_1}$
Hubble's Constant: $H_0 = \dfrac{1}{R(t_0)} \dfrac{dR(t_0)}{dt}$
Deceleration constant: $q_0 = -\dfrac{1}{RH_0^2} \dfrac{d^2R}{dt^2}$
Hubble's Law: $H_0 d_L = z + \frac{1}{2}(1 - q_0)z^2 + \cdots$
Galaxy count-Red Shift Relation: $\dfrac{1}{z^2} \dfrac{dN_{gal}}{dz d\Omega} = (H_0 R_0)^{-3} n_c(z)[1 - 2(q_0 + 1)z + \cdots]$
Angular diameter-Red Shift Relation: $H_0 d_A = z - \dfrac{1}{2}(3 + q_0)z^2 + \cdots$

2.2.2 Standard Cosmology

In the Standard cosmology,the well-known **Einstein equation** from the General Theory of Relativity is,

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = G_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu} \tag{2.2}$$

where $G_{\mu\nu}$ is the Einstein tensor,$T_{\mu\nu}$ is the Field-Energy tensor for all the fields present and Λ is the cosmological constant.
Einstein-Hilbert Action: $S_{E-H} = -\dfrac{1}{16\pi G} \int d^4x \sqrt{-g}(R + 2\Lambda)$
Matter Action: $S_M = \sum_{fields} \int d^4x \sqrt{-g} L_{fields}$
The relation between energy density and R can be summarized as,
For a simple equation of state,$p = w\rho$,so then if we consider w is independent of time,then

$$\rho \propto R^{-3(1+w)}$$

$$RADIATION \Rightarrow (p = \frac{1}{3}\rho), then \rho \propto R^{-4} \tag{2.3}$$

$$MATTER \Rightarrow (p = 0), then \rho \propto R^{-3} \tag{2.4}$$

$$VACUUMENERGY \Rightarrow (p = -\rho), then \rho = const. \tag{2.5}$$

From the zeroth component of the tensorial quantities,we can get the **Friedmann equation**,as

$$\frac{1}{R^2}(\frac{dR}{dt})^2 + \frac{k}{R^2} = \frac{8\pi G}{3}\rho$$

Recasting Friedmann equation,

$$\frac{k}{H^2 R^2} = \Omega - 1$$

where, $\Omega = \frac{\rho}{\rho_c}$ and $\rho_c = \frac{3H^2}{8\pi G}$

Expansion Age of The Universe

$$(\frac{1}{R_0}\frac{dR}{dt})^2 + \frac{k}{R_0^2} = \frac{8\pi G}{3}\rho_0\frac{R_0}{R} \tag{2.6}$$

$$(\frac{1}{R_0}\frac{dR}{dt})^2 + \frac{k}{R_0^2} = \frac{8\pi G}{3}\rho_0(\frac{R_0}{R})^2 \tag{2.7}$$

2.3 Equilibrium Thermodynamics

Today the radiation,or relativistic particles,in the universe is comprised of the 2.75K microwave photons,and the 3 cosmic seas of 1.96K relic neutrinos.

$$Numberdensity, n = \frac{g}{(2\pi)^3}\int f(\overrightarrow{p})d^3p \tag{2.8}$$

$$Energydensity, \rho = \frac{g}{(2\pi)^3}\int f(\overrightarrow{p})E(\overrightarrow{p})d^3p \tag{2.9}$$

$$Pressure, p = \frac{g}{(2\pi)^3}\int f(\overrightarrow{p})\frac{(|\overrightarrow{p}|)^2}{3E}d^3p \tag{2.10}$$

where, $f(\overrightarrow{p}) = [exp(\frac{E-\mu}{T}) \pm 1]^{-1}$ is the familiar Fermi-Dirac and Bose-Einstein distribution function.
In the relativistic limit,i.e, $T \gg m$,
$\rho = \frac{\pi^2}{30}gT^4(forBosons)$ and $\rho = \frac{7}{8}\frac{\pi^2}{30}gT^4(forFermions)$
$n = \frac{\varsigma(3)}{\pi^2}gT^3(forBosons)$ and $n = \frac{3}{4}\frac{\varsigma(3)}{\pi^2}gT^3(forFermions)$
$p = \frac{\rho}{3}$
For degenerate Fermions,
$\rho = \frac{1}{8\pi^2}g\mu^4$
$n = \frac{1}{6\pi^2}g\mu^3$
$p = \frac{1}{24\pi^2}g\mu^4$
For relativistic Bosons or fermions for which $|\mu| \ll T$
$n = exp(\frac{\mu}{T})\frac{g}{\pi^2}T^3$
$\rho = exp(\frac{\mu}{T})\frac{3g}{\pi^2}T^4$
$n = exp(\frac{\mu}{T})\frac{g}{\pi^2}T^4$
In non-relativistic limit,i.e, $m \gg T$
$n = g(\frac{mT}{2\pi})^{\frac{3}{2}}exp[-\frac{m-\mu}{T}]$
$\rho = mn$
$p = nT \ll \rho$

If the reaction $particle + antiparticle \leftrightarrow \gamma + \gamma$ are occuring rapidly then the net fermion number will be,
$n_+ - n_- = \frac{gT^3}{6\pi^2}[\pi^2(\frac{\mu}{T}) + (\frac{\mu}{T})^3](forT \gg m)$ and
$n_+ - n_- = 2g(\frac{mT}{2\pi})^{\frac{3}{2}} sinh(\frac{\mu}{T}) exp(-\frac{\mu}{T})(forT \ll m)$
The total energy density and pressure of all species in thermal equilibrium is,

$$\rho_R = T^4 \sum_{i=allspecies} (\frac{T_i}{T})^4 \frac{g_i}{2\pi^2} \int_{x_i}^{\infty} \frac{(u^2 - x_i^2)^{\frac{1}{2}} u^2 du}{exp(u - y_i) \pm 1}$$

$$p_R = T^4 \sum_{i=allspecies} (\frac{T_i}{T})^4 \frac{g_i}{6\pi^2} \int_{x_i}^{\infty} \frac{(u^2 - x_i^2)^{\frac{3}{2}} du}{exp(u - y_i) \pm 1}$$

where $x_i = \frac{m_i}{T}$ and $y_i = \frac{\mu_i}{T}$
For relativistic species,

$$\rho_R = \frac{\pi^2}{30} g_* T^4$$

and

$$p_R = \frac{\pi^2}{90} g_* T^4$$

where $g_* =$ total no of effective massless degrees of freedom $= \sum_{i=bosons} g_i (\frac{T_i}{T})^4 + \frac{7}{8} \sum_{i=fermions} g_i (\frac{T_i}{T})^4$
During the early radiation-dominated epoch,

$$H = 1.66 g_*^{\frac{1}{2}} \frac{T^2}{m_{pl}}$$

and

$$t = 0.301 g_*^{-\frac{1}{2}} \frac{m_{pl}}{T^2}$$

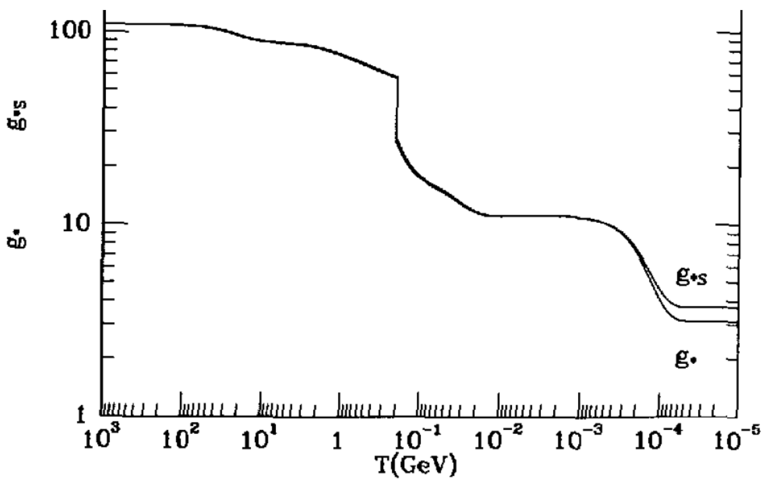


Figure 2.1: The evolution of $g_*(T)$ as a function of temperature in the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$

3

2.3.1 Entropy

In thermal equilibrium,the entropy per comoving volume,S,is constant i.e,$d\{\frac{(\rho+p)V}{T}\}=0$ where we want to introduce a new quantity,entropy density

$$s=\frac{S}{V}$$

The entropy density is dominated by the relativistic particles,so to a good approximation,we get,

$$s=\frac{2\pi^2}{45}g_{*s}T^3$$

where,$g_{*s}=g_{*}*(\frac{T}{T_i})$
Conservation of entropy implies that,$s\propto R^{-3}$,Number of some species in comoving volume is equal to number density divided by the entropy density $=N=R^3n=\frac{n}{s}$
In thermal equilibrium,we can get

$$N=\frac{45\varsigma(3)g}{2\pi^4 g_{*}s}\text{ for }T\gg m,\mu$$

$$N=\frac{45g}{4\sqrt{2}\pi^5 g_{*s}}(\frac{m}{T})^{\frac{3}{2}}exp(-\frac{m}{T}+\frac{\mu}{T})\text{ for }T\ll m$$

Temperature of universe evolves as

$$T\propto g_{*s}^{-\frac{1}{3}}R^{-1}$$

Distribution function for a massless particle species remains self-similar as the universe expands,with the temperature red-shifting as,

$$T=T_D\frac{R_D}{R}\propto R^{-1}$$

Distribution function for a massive particle species remains self-similar as the universe expands,with the temperature red-shifting as,

$$T=T_D(\frac{R_D}{R})^2\propto R^{-2}$$

Again some important facts are there as,$\frac{T}{T_\nu}=(\frac{11}{4})^{\frac{1}{3}}=1.40$ (today)
Again for today's universe we get that, $g_{*}=3.36$ and $g_{*s}=3.91$
Not only that,the other physical quantities are, $\rho_R=8.09*10^{-34}$ **g** cm^{-3},$\Omega_R h^2=4.31*10^{-5}$,$s\simeq 2970cm^{-3}$ **and** $n_\gamma=422cm^{-3}$

2.4 Thermodynamics in The Expanding Universe

2.4.1 The Boltzmann Equation

To a good approximation,we can say that the most of the ingredients of the early universe are in thermal equilibrium.But,some notable departures from the equilibrium conditions i.e,neutrino decoupling,decoupling of background radiation,Primordial nucleosynthesis,inflation,baryogenesis,Decoupling of relic WIMPs etc.

The criterion of any species to be coupled or decoupled involves the comparison of the interaction rate of the particle Γ and the expansion rate of the Universe H i.e,

$$\Gamma \geq H \text{ (coupled)}$$

$$\Gamma \leq H \text{ (decoupled)}$$

Boltzmann equation can be written as

$$\widehat{L}[f] = C[f]$$

where $\widehat{L}$ is the Liouville operator,C is the collision operator.

The Liouville operator in non-relativistic form is,$\widehat{L}_{NR} = \frac{d}{dt} + \overrightarrow{v}.\overrightarrow{\nabla}_x + \frac{\overrightarrow{F}}{m}.\overrightarrow{\nabla}_v$ The covariant,relativistic generalisation of the Liouville operator is

$$\widehat{L} = p^\alpha \frac{\partial}{\partial x_\alpha} - \Gamma^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial}{\partial p^\alpha}$$

At last Boltzmann-equation can be written as in the form,

$$\frac{dn}{dt} + \frac{3n}{R}\frac{dR}{dt} = \frac{g}{(2\pi)^3}\int C[f]\frac{d^3p}{E} \tag{2.11}$$

The collision term of the process,$\psi + a + b + \cdots\cdots \leftrightarrow i + j + \cdots\cdots$ we get that,

$$\begin{aligned}\frac{g}{(2\pi)^3}\int C[f]\frac{d^3p_\psi}{E_\psi} = -\int d\pi_\psi d\pi_a d\pi_b \cdots d\pi_i d\pi_j \cdots (2\pi)^4 \delta^4(P_\psi + P_a + P_b \cdots - P_i - P_j - \cdots)\\ [(|M|)^2_{\psi+a+b\cdots\rightarrow i+j+\cdots} f_a f_b \cdots\cdots f_\psi (1 \pm f_i)(1 \pm f_j)\cdots\\ - (|M|)^2_{i+j+\cdots\rightarrow\psi+a+b+\cdots} f_i f_j \cdots\cdots (1 \pm f_a)(1 \pm f_b)\cdots(1 \pm f_\psi)]\end{aligned}$$

where f_i is the phase-space densities of the i-th species and $d\pi = \frac{g}{(2\pi)^3}\frac{d^3p}{2E}$
There are two well-motivated approximations,i.e,one is T or CP invariance of the $(|M|)^2$ term and the second assumption is the use of Maxwell-Boltzmann Statistics instead of Fermi-Dirac or Bose-Einstein Statistics.So we can write the Bolzmann equation in more familiar form as,

$$\frac{dn_\psi}{dt} + 3Hn_\psi = -\int d\pi_\psi d\pi_a \cdots d\pi_i d\pi_j \cdots (2\pi)^4 (|M|)^2 \delta^4(P_i + P_j \cdots - P_\psi - P_a - \cdots)[f_a f_b \cdots - f_i f_j \cdots] \tag{2.12}$$

If we define the relic abundance as $Y = \frac{n_\psi}{s}$,then we can write the Boltzmann-equation as,

$$\frac{dn_\psi}{dt} + 3Hn_\psi = s\frac{dY}{dt} \tag{2.13}$$

Introducing the independent variable,$x = \frac{m}{T}$ which we can call as relativistic-non-relativistic measure.

2.4.2 Freeze Out:Origin of Species

If we want the creation and annihilation process in the equation,$\psi\overline{\psi} \leftrightarrow X\overline{X}$

$$\frac{dY}{dx} = \frac{-x\langle\sigma_{\psi\overline{\psi}\leftrightarrow X\overline{X}}|v|\rangle s}{H(m)}(Y^2 - Y^2_{EQ}) \tag{2.14}$$

So,here thermally averaged cross-section times velocity is actually,

$$\langle\sigma_{\psi\overline{\psi}\leftrightarrow X\overline{X}}|v|\rangle = (n^{EQ}_\psi)^{-2}\int d\pi_\psi d\pi_{\overline{\psi}} d\pi_X d\pi_{\overline{X}} (2\pi)^4 \delta^4(P_\psi + P_{\overline{\psi}} - P_X - P_{\overline{X}})(|M|)^2 exp(-\frac{E_\psi}{T})exp(-\frac{E_{\overline{\psi}}}{T}) \tag{2.15}$$

According to relativistic and non-relativistic regime,we can get two different relics there,

Hot Relics

In this case freeze-out occurs when the species is still relativistic($x_f \leq 3$) and the Y_{EQ} is not changing with time.As the Y_{EQ} is constant with time,so the final value of Y is very much insensitive to the details of freeze-out.The equilibrium value of freeze-out is given by,

$$Y_\infty = Y_{EQ}(x_f) = 0.278\frac{g_{eff}}{g_{*s}(x_f)}$$

The abundance of ψ's today is $n_{\psi 0} = 2970 Y_\infty cm^{-3}$ Again,from here we get that,$\rho_{\psi 0} = 2.97 * 10^3 Y_\infty (\frac{m}{eV})$ eV cm^{-3} and

$$\Omega_\psi h^2 = 7.83 * 10^{-2}[\frac{g_{eff}}{g_{*s}(x_f)}](\frac{m}{eV})$$

From here we can get that,$m_\nu \leq 91.5$ eV,so this cosmological bound for the stable,light neutrino species is often referred as **Cowsik-McClelland bound**.

Examples of hot relic species include light photino and light gravitino etc.

Cold Relics

It is the more difficult case than the previous one,here freeze-out occurs when the particle species becomes non-relativistic($x_f \geq 3$) and Y_{EQ} is decreasing exponentially with x.Here the Boltzmann-equation becomes,

$$\frac{dY}{dx} = -\lambda x^{-n-2}(Y^2 - Y_{EQ}^2)$$

where,

$$\lambda = 0.264(\frac{g_{*s}}{g_*^{\frac{1}{2}}}) m_{pl} m \sigma_0$$

$$Y_{EQ} = 0.145(\frac{g}{g_{*s}}) x^{\frac{3}{2}} e^{-x}$$

From here,after solving the integration perfectly,we can find that,

$$x_f = ln[0.038(n+1)(\frac{g}{g_*^{\frac{1}{2}}}) m_{pl} m \sigma_0] - (n + \frac{1}{2}) lnln[0.038(n+1)(\frac{g}{g_*^{\frac{1}{2}}}) m_{pl} m \sigma_0] \tag{2.16}$$

$$Y_\infty = \frac{3.79(n+1) x_f^{n+1}}{(\frac{g_{*s}}{g_*^{\frac{1}{2}}}) m_{pl} m \sigma_0} \tag{2.17}$$

$$\Omega_\psi h^2 = 1.07 * 10^9 \frac{(n+1) x_f^{n+1} GeV^{-1}}{(\frac{g_{*s}}{g_*^{\frac{1}{2}}}) m_{pl} \sigma_0} \tag{2.18}$$

I use these three equations 16,17,18 for the calculation of the project of mine which is discussed in the next chapter.

From there,we can calculate that,$m_\nu \geq 2$GeV,It is known as **Lee-Weinberg Bound**.

Chapter 3

ITM for the Scalar Dark Matter

3.1 Introduction

There are strong evidences for the non-baryonic Dark Matter which according to Planck satellite constitute more than 0.26 of energy density in the universe.WIMP's as a relic remnants of the early universe are the most plausible candidates for the Dark Matter.Since the Standard Model cannot explain Dark Matter evidences,there is a strong motivation to extend Standard Model in a way to provide suitable Dark matter candidate.Singlet scalar or Fermion fields are preferred as simple candidates of Dark Matter.So,here one of the simplest models for a scalar dark matter is ITM(Inert Triplet Model).In this model,a scalar $SU(2)_L$ triplet is odd under Z_2 symmetry so that they can directly couple to the SM particles and the neutral components of the triplets play role of Dark Matter.

After a few decades of expectations,the LHC has found a Standard Model like Higgs Particle with a mass of 125 GeV.Since the Higgs boson can participate in DM-nucleon scattering and DM annihilation,current analysis of the LHC data and measurements of its decay rates would set limit on any beyond SM that provides a DM candidate.

In this chapter,I shall review Inert Triplet Model (ITM) which provide candidate for Dark Matter particles and so on.

In this project we discuss about the Inert Scalar Triplet Model to explain the possible Dark Matter candidate stabilized by Z_2 extension of the Standard Model with the constraints of the constants.Here we determine the range of possible mass in the suitable range of the current relic density of the dark matter in the present universe.

To determine the range of the mass in this model,we can proceed in this following way,which is followed by some steps discussed below,

1.At first,we determine some dark-matter particle self annihilation processes from the Lagrangian depicted in this model.

2.Then,we determine the cross-section of the all possible processes,and then sum them all up to determine the total cross-section of the all scattering processes.

3.Then,from there we can get the relic density of the dark matter particle using the 16,17,18 equations discussed in the last chapter.

4.from there,we can get the relation between the relic density and the mass of the Dark Matter particle species,then after matching the relic density range,we can determine the mass range of the possible dark matter species.

3.2 About the model

In ITM,the matter content of SM is extended with a $SU(2)_L$ triplet scalar with the hypercharge $Y = 0$ or $Y = 2$.These additional fields are odd under Z_2 symmetry condition while all the SM fields own even eigenvalues.The Z_2 symmetry is not spontaneously broken since the triplet does not develop a vacuum expectation value.The triplet T for $Y = 0$ has $VEV = 0$ and the SM Higgs doublet H and the triplet T scalars are defined as,

$$T = \begin{pmatrix} \frac{1}{\sqrt{2}}T^0 & -T^+ \\ -T^- & -\frac{1}{\sqrt{2}}T^0 \end{pmatrix} \tag{3.1}$$

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{3.2}$$

where $v = 246GeV$.
The relevant Lagrangian which is allowed by Z_2 symmetry can be given by:

$$L = |D_\mu H|^2 + tr|D_\mu T|^2 - V(H,T) \tag{3.3}$$

and

$$V(H,T) = m^2|H|^2 + M^2 tr[T^2] + \lambda_1|H|^4 + \lambda_2(tr[T^2])^2 + \lambda_3|H|^2 tr[T^2] \tag{3.4}$$

In this case $Y = 0$,ITM has three new parameters compared to the Standard Model.We require that Higgs potential is bounded from the below,which leads to the following conditions on the parameters of the potential:

$$\lambda_1, \lambda_2 \geq 0, (\lambda_1\lambda_2)^{\frac{1}{2}} - \frac{1}{2}|\lambda_3| > 0 \tag{3.5}$$

The conditions for the local minimum are satisfied if and only if $m^2 < 0$,$v^2 = -\frac{m^2}{2\lambda_1}$ and $2M^2 + \lambda_3 v^2 > 0$.The masses of triplet scalars can be written as,

$$m_{T^0} = m_{T^\pm} = \sqrt{M^2 + \frac{1}{2}\lambda_3 v^2} \tag{3.6}$$

Here,we can note that at the tree level,masses of neutral and charged components are the same,but at the loop level the $T^\pm$ are slightly heavier than T^0.The scalar and gauge interactions of ITM have been extracted in terms in terms of real fields.In case $Y = 0$,the Z_2 symmetry ensures that the T^0 can decay to SM fermions and can be considered as cold DM candidate.

In case $Y = 2$ the $SU(2)_L$ triplet can be parametrized with the five new parameters.But it is excluded in my project.The ITM with $Y = 2$ is already excluded by the limits from direct detection experiments.So,there won't be any use to study the case in this regard.

Here,in the Relic density calculation I can say that,the relic density of DM is well measured by WMAP and Plancks's experiments and the current value is :

$$\Omega_{DM}h^2 = 0.1199 \pm 0.0027$$

where I am using the value,

$$h = 0.67 \pm 0.012$$

It is the scaled current Hubble parameter in units of 100km/s.Mpc.In,the following,i am using this value as upper bound on the contribution of ITM in production of DM.

3.3 From the Lagrangian

From the Lagrangian,we can get,

$$m^2|H|^2 = m^2H^+H \tag{3.7}$$

$$M^2tr[T^2] = M^2[2[T^+]^2 + (T^0)^2] \tag{3.8}$$

$$\lambda_1|H|^4 = \lambda_1(H^+)^2H^2 \tag{3.9}$$

$$\lambda_2(tr[T^2])^2 = \lambda_2[2[T^+]^2 + (T^0)^2]^2 \tag{3.10}$$

$$\lambda_3|H|^2tr[T^2] = \lambda_3(H^+H)[2[T^+]^2 + (T^0)^2] \tag{3.11}$$

Not only that,we can define the terms as,

$$D_\mu = \partial_\mu - ig[W^a_\mu \frac{\sigma^a}{2}] - ig\prime\frac{Y}{2}B_\mu \tag{3.12}$$

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu) \tag{3.13}$$

$$Z_\mu = c_W W^3_\mu - s_W B_\mu \tag{3.14}$$

$$A_\mu = s_W W^3_\mu + c_W B_\mu \tag{3.15}$$

After a big calculation on the tr $|D_\mu T|^2$ in the Lagrangian using the upper-mentioned definitions we can get the final expression as,

$$tr(|D_\mu T|)^2 = (\partial_\mu T_0)^2 - \frac{g\prime^2}{4}(c_W Z_\mu + s_W A_\mu)^2T_0^2 - \frac{g^2}{4}(c_W Z_\mu - s_W A_\mu)^2T_0^2 - \frac{g^2}{4}(W^+_\mu)^2T_0^2 - \frac{g^2}{4}(W^-_\mu)^2T_0^2 \tag{3.16}$$

Here,we can say that from the all terms with the $T^0 - T^0$ self-interactions,the possible scattering processes are 2-four point gauge interactions as T^0T^0 go to $Z_\mu Z_\nu$ and T^0T^0 go to $W^+_\mu W^+_\mu$ as well as two scattering processes as T^0T^0 go to standard model fermion-antifermion pair via Z-boson and Higgs exchange.

3.4 Possible Feynman Diagrams

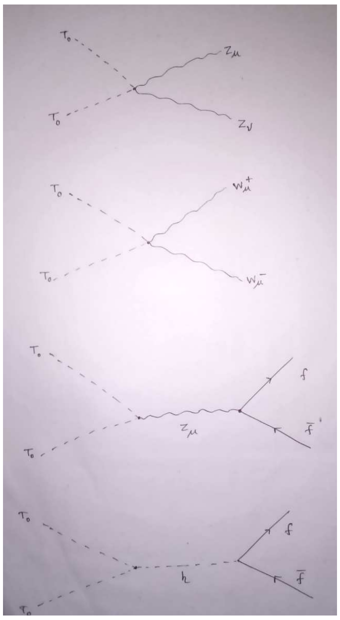


Figure 3.1: Possible Feynman Diagrams

3.5 Calculation of the Scattering cross-sections

3.5.1 Some important formulae to calculate cross-section

Here for the determination of scattering-cross-section we can get,

$$d\sigma = \frac{1}{2E_A 2E_B |v_A - v_B|}(\Pi_f \frac{d^3P_f}{(2\pi)^3}\frac{1}{2E_f})|M|^2(2\pi)^4\delta^4(P_A + P_B - \Sigma P_f) \tag{3.17}$$

$$\int d\pi_a = (\Pi_f \frac{d^3P_f}{(2\pi)^3}\frac{1}{2E_f})(2\pi)^4\delta^4(P - \Sigma P_f) \tag{3.18}$$

For the special case of two particles in the final state,we can simplify the expression by partially evaluating the phase-space integrals in the C.M frame,

$$\int d\pi_2 = \int \frac{dP_1 P_1^2 d\Omega}{(2\pi)^3 2E_1 2E_2}(2\pi)\delta(E_{cm} - E_1 - E_2) = \int d\Omega \frac{1}{16\pi^2}\frac{|P_1|}{E_{cm}} \tag{3.19}$$

For reactions symmetric about the collision axis,

$$\int d\pi_2 = \int d(cos\theta)\frac{1}{16\pi}\frac{2|P_1|}{E_{cm}} \tag{3.20}$$

At last,we can say,the differential cross-section becomes,

$$(\frac{d\sigma}{d\Omega})_{CM} = \frac{1}{2E_A 2E_B|v_A - v_B|}\frac{|P_1|}{(2\pi)^2 4E_{cm}}|M|^2 \tag{3.21}$$

and here,

$$|P_1| = \frac{E_A + E_B}{2}(1 - \frac{m_{f_1}^2 + m_{f_2}^2}{(E_A + E_B)^2}) \tag{3.22}$$

Then,by integrating over the angular variables θ and ϕ we can get the required **scattering cross-section.**

3.5.2 $T^0T^0 \leftrightarrow f + \overline{f}$ via Z-boson exchange ⇒

$$iM = \{g\frac{cos2\theta_W}{2cos\theta_W}(P_A - P_B)^{\mu}\}\{\frac{-ig_{\mu\nu}}{k^2 - M_Z^2}\}[\overline{u}\frac{ig}{cos\theta_W}\gamma^{\nu}(g_V^f - g_A^f\gamma_5)v] \tag{3.23}$$

then,

$$\overline{|M|}^2 = \frac{g^4 cos^2 2\theta_W}{4cos^4\theta_W}(P_A - P_B)^{\mu}(P_A - P_B)^{\rho}\{\frac{-ig_{\mu\nu}}{k^2 - M_Z^2}\frac{-ig_{\rho\sigma}}{k^2 - M_Z^2}\}(\overline{u}\gamma^{\nu}(g_V^f - g_A^f\gamma_5)v)(\overline{u}\gamma^{\sigma}(g_V^f - g_A^f\gamma_5)v)^{+} \tag{3.24}$$

From there we can get,with the help of trace calculation,in the non-relativistic limit,

$$\langle\sigma|v|\rangle = \frac{3g^4cos^2 2\theta_W}{32\pi^2 cos^4\theta_W x_f}\frac{(g_V^f)^2 + (g_A^f)^2}{(4M_{DM}^2 - M_Z^2)^2}(M_f^2 - M_{DM}^2)(1 - \frac{M_f^2}{2M_{DM}^2})^{\frac{1}{2}} \tag{3.25}$$

here,we can consider fermion f as all the leptons i.e,electron,muon,tauon and their corresponding neutrinos as well as all the quarks i.e,up,down,charm,strange,top and bottom quarks.

3.5.3 $T^0T^0 \leftrightarrow f + \overline{f}$ via Higgs exchange ⇒

$$iM = -\frac{i}{2}g\frac{M_h^2}{M_W}\{\frac{i}{P^2 - M_h^2}\}(-i\frac{g}{2}\frac{M_f}{M_W}) \tag{3.26}$$

then,

$$\overline{|M|}^2 = \frac{1}{16}g^4\frac{M_h^4 M_f^2}{M_W^4(P^2 - M_h^2)^2} \tag{3.27}$$

From there we can get,with the help of trace calculation,in the non-relativistic limit,

$$\langle\sigma|v|\rangle = \frac{1}{2048\pi^2 M_{DM}^3}(4M_{DM}^2 - 2M_f^2)^{\frac{1}{2}}g^4\frac{M_h^4 M_f^2}{M_W^4(4M_{DM}^2 - M_h^2)^2} \tag{3.28}$$

3.5.4 $T^0T^0 \leftrightarrow Z_{\mu}Z_{\nu}$ ⇒

$$iM = \{\frac{i}{2}(\frac{gcos2\theta_W}{cos\theta_W})^2 g_{\mu\nu}\}\epsilon^{\mu}\epsilon^{*\nu} \tag{3.29}$$

then,

$$\overline{|M|}^2 = \frac{1}{4}(\frac{gcos2\theta_W}{cos\theta_W})^4[16 + 8\frac{k^2}{M_W^2} + \frac{k^4}{M_W^4}] \tag{3.30}$$

From there we can get,with the help of trace calculation,in the non-relativistic limit,

$$\langle\sigma|v|\rangle = \frac{1}{256\sqrt{2}\pi^2 M_{DM}^3}(2M_{DM}^2 - M_Z^2)^{\frac{1}{2}}(\frac{gcos2\theta_W}{cos\theta_W})^4[16 + 8\frac{k^2}{M_W^2} + \frac{k^4}{M_W^4}] \tag{3.31}$$

3.5.5 $T^0T^0 \leftrightarrow W_\mu^+ W_\nu^- \Rightarrow$

$$iM = \{\frac{i}{2}g^2 g_{\mu\nu}\}\epsilon'^{\mu}\epsilon^{*\nu} \tag{3.32}$$

then,

$$\overline{|M|}^2 = \frac{25}{4}g^4 \tag{3.33}$$

From there we can get,with the help of trace calculation,in the non-relativistic limit,

$$\langle\sigma|v|\rangle = \frac{25g^4}{512M_{DM}^3\pi^2}(4M_{DM}^2 - 2M_W^2)^{\frac{1}{2}} \tag{3.34}$$

Now,I can plot some graphs individually as well as combiningly,then we get some plots,which are given below.And the significance of the plots are discussed in the "Conclusion" part.

Chapter 4

Results,Plots and Conclusions

4.1 Introduction

In this chapter,we summarize all the discussions and work done so far in this project about the calculation of the correct mass range of the possible scalar dark matter according to the range of correct relic density of the dark matter in the present day universe.Then,we discuss some modifications on the work done so that the work done so far can be done more preciously and also discuss the significance of the plots given in the 3rd chapter under the section ”Plots” coming from the gnuplot here.

4.2 Plots

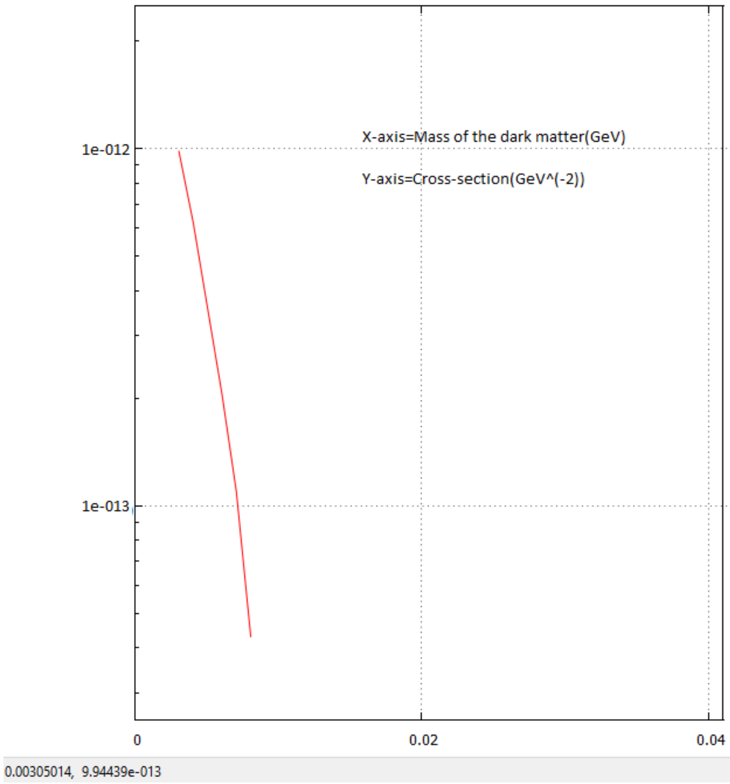


Figure 4.1: u-quark contribution

For the plots 1 to 10,we get the plots for the individual contribution of the fermions i.e,leptons and quarks.Here actually we know that,from Standard Model we know that the masses of the fermions are ranging a lot.It can be listed below,

$M_u = 0.0024, M_d = 0.0048, M_s = 0.104, M_c = 1.27, M_b = 4.2, M_t = 174.0, M_{electron} = 0.0005, M_{muon} = 0.106, M_{tauon} = 1.8, M_{\nu_e} = 0.0, M_{\nu_\mu} = 0.00017, M_{\nu_\tau} = 0.015$

Here,all are expressed in the GeV unit.So,from there we also note that,from the two terms $(1-\frac{M_f^2}{2M_{DM}^2})^{\frac{1}{2}}$ as well as $(4M_{DM}^2 - 2M_f^2)^{\frac{1}{2}}$ appearing in the cross-section calculation of the first two processes,so from there we get,for a real decay process to be held the following condition must be satisfied by the Dark matter mass as well as Fermion mass i.e,

$$M_{DM} \geqslant \frac{M_f}{\sqrt{2}}$$

So,we get that,all the fermion channels are not opened simultaneously.They are opened with respect to their consecutive masses and after that sometime comes,when they are dominated by the greater mass fermions.These type of behaviour we can see from the combined plots from 11 to 14 plots.

This is the plot of the total cross-section times the thermally-averaged relative velocity coming from the all possible feynman diagrams vs the mass of the scalar dark matter setting logscale in the y-axis i.e,in the cross-section axis.It is a very common curve showing the behaviour that,the cross-section is inversely proportional to the relic density i.e, $\Omega_{DM}h^2$

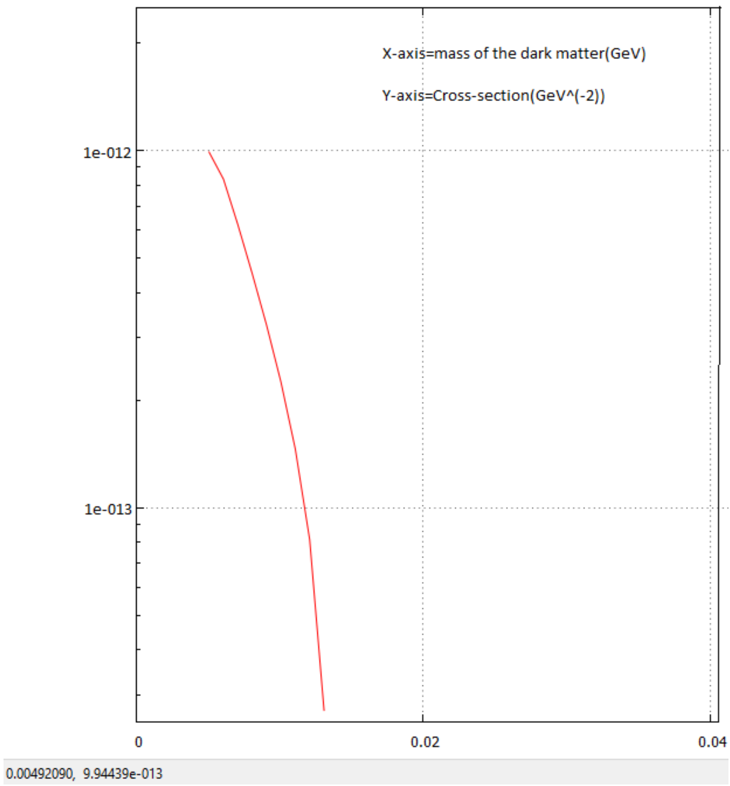


Figure 4.2: d-quark contribution

There is a lot to say about these plots i.e,from 4.16 to 4.18.Actually the plot is plotted between the total relic density of the Dark Matter in the present universe and the mass of the Dark matter.Here the first plot is plotted with the y value ranging from **0.0001** to **1**. And as the range of the relic density differs from the value from **0.1176** to **0.1226** so,I am choosing in the 2nd plot the y value ranging from **0.1** to **1** from the more clearer view.This is the most important graph in my project work.

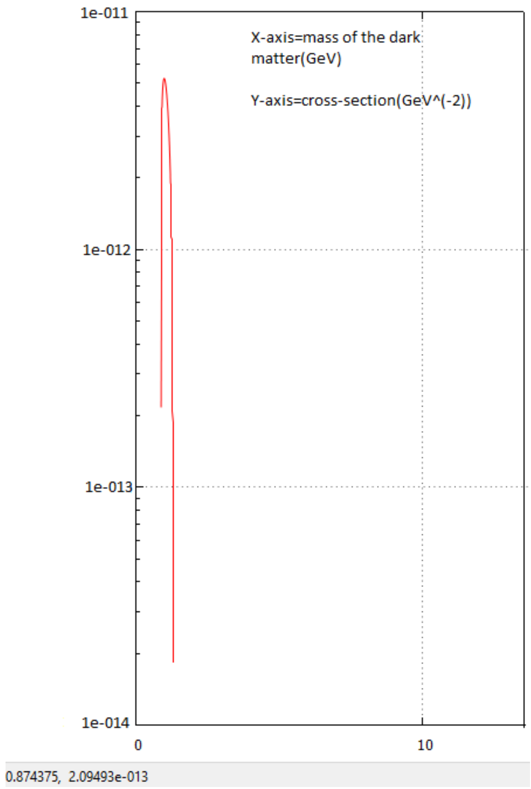


Figure 4.3: c-quark contribution

So,from this figure we can easily say that,the mass range of the dark matter ranging from approximately **8220 GeV** to **8420 GeV** according to my possible feynman diagrams.

4.3 Future aspects

After the project completion,we want to say that some modifications should be made to get the better result in this context.These are follows consequently,

1.we are using for the calculation of the Feynman amplitude as well as the scattering cross-section times the thermally averaged relative velocity the notions of triple Higgs and goldstone interactions in stead of all the pure scalar triplet and Higgs or Z-boson interaction.They are not the same,but we can use that so that they can produce approximately the same answer.

2.Here we are also using values of the coupling constants from the standard table,if there is discrepancy from the real value,then some modifications should be made for the better result.

3.Gnuplot cannot combine all the values for the programming with so many points,so if more efficient compiler as well as plotting programme can be used then it is more efficient to determine the actual values of the mass range.

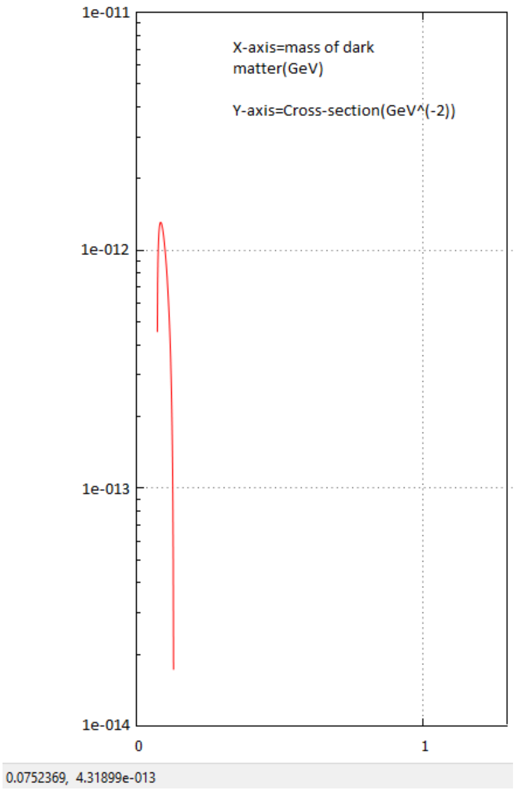


Figure 4.4: s-quark contribution

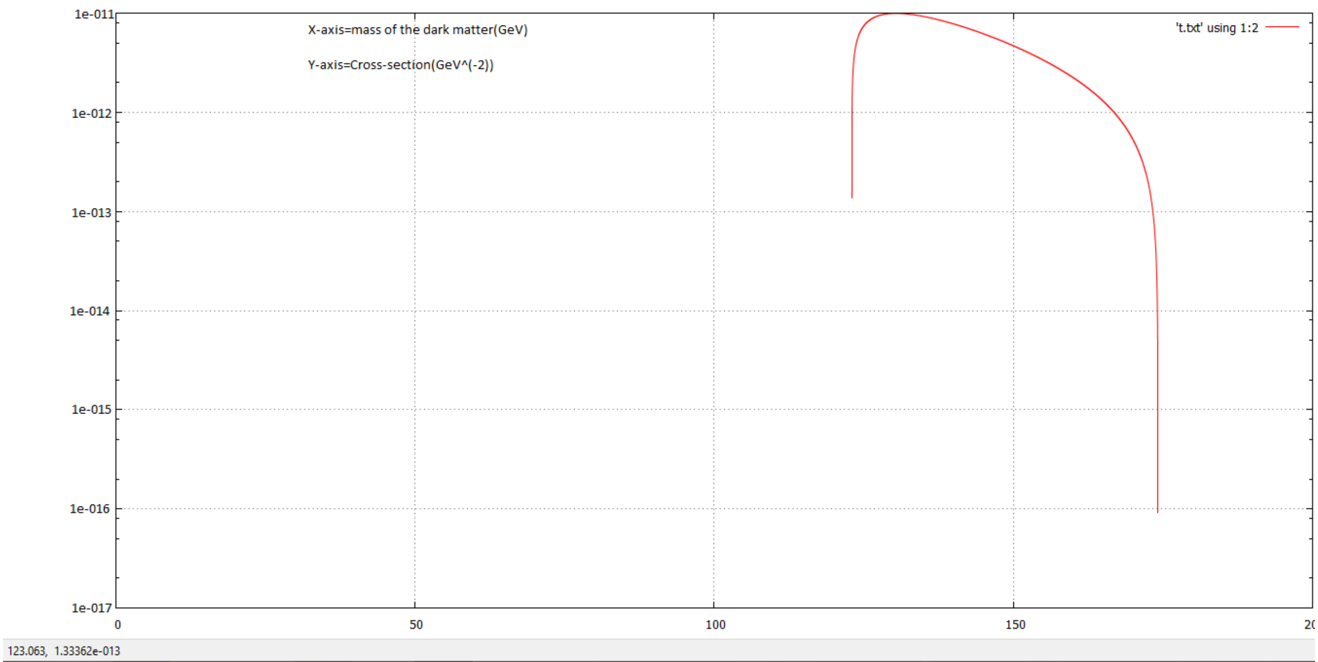


Figure 4.5: t-quark contribution

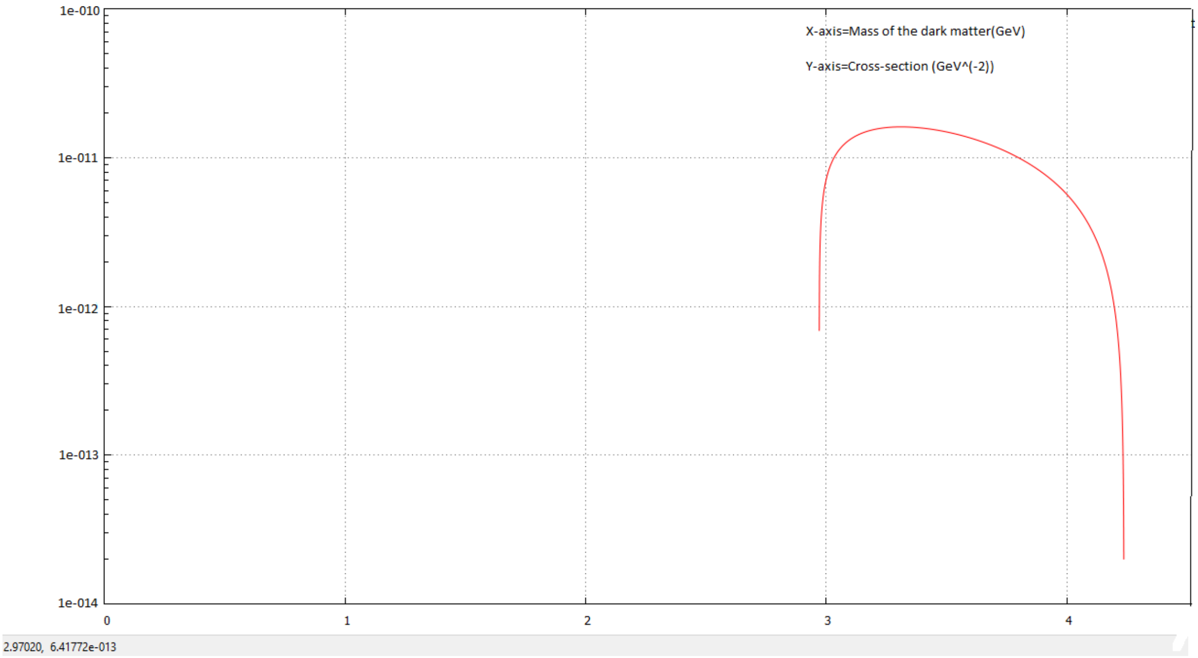


Figure 4.6: b-quark contribution

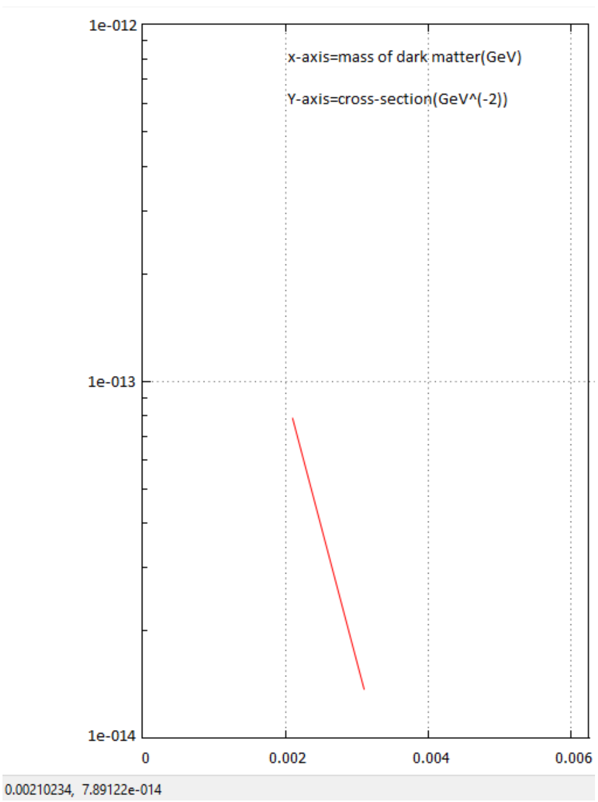


Figure 4.7: electron contribution

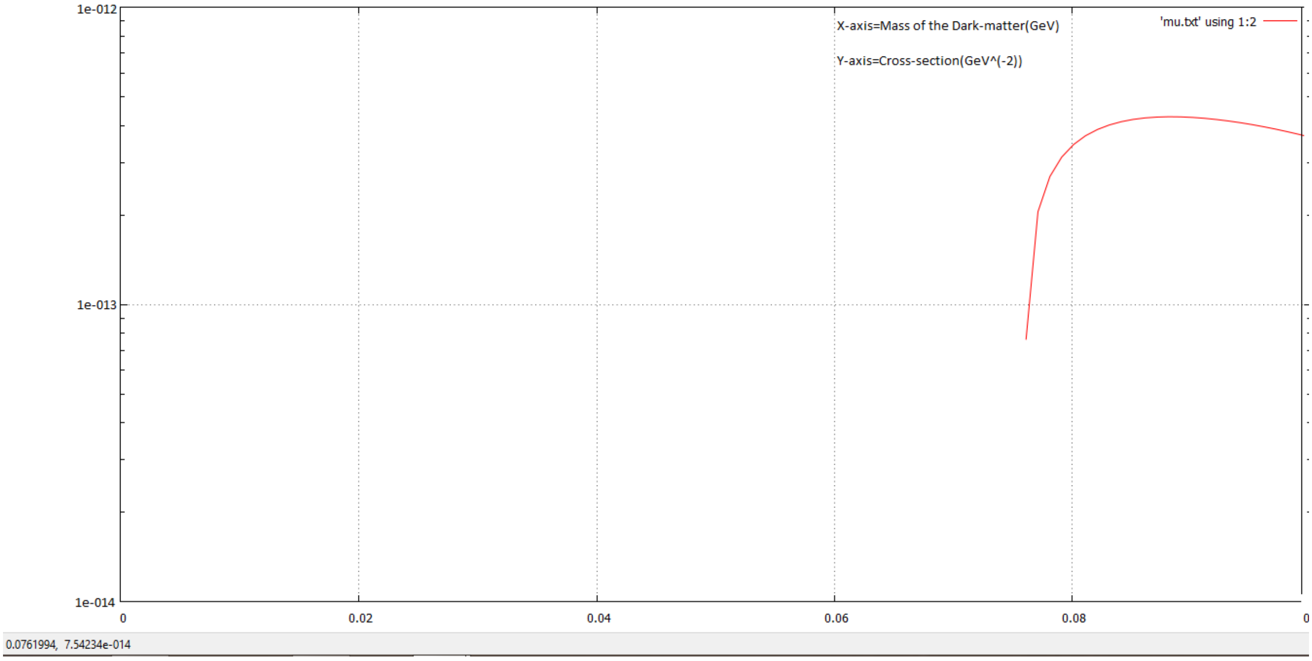


Figure 4.8: muon contribution

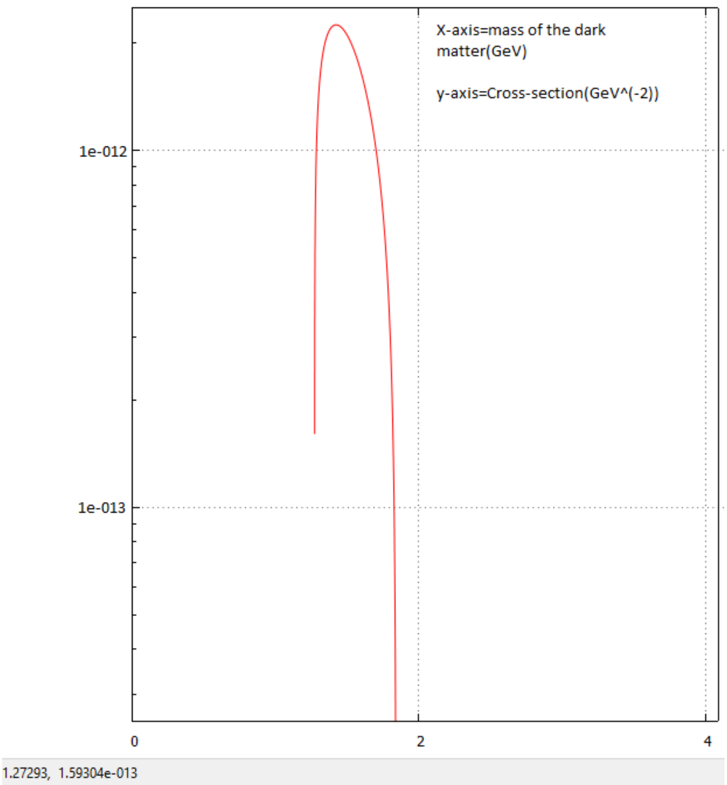


Figure 4.9: tauon contribution

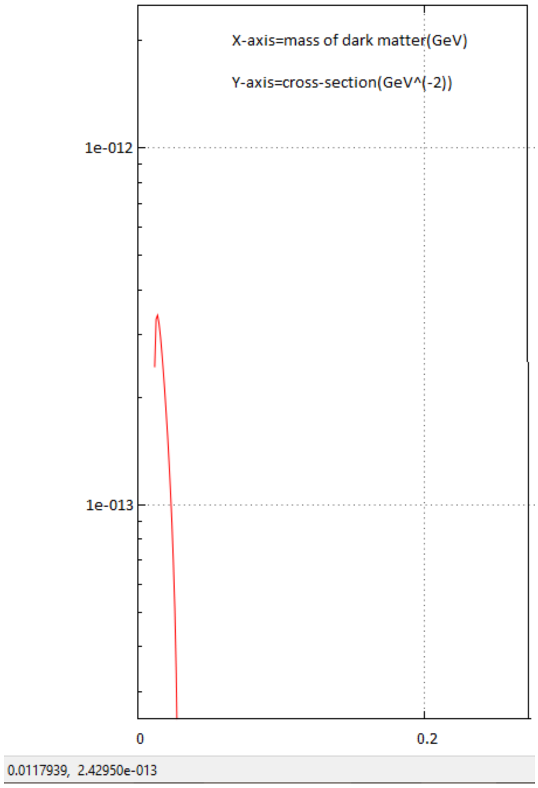


Figure 4.10: tau-neutrino contribution

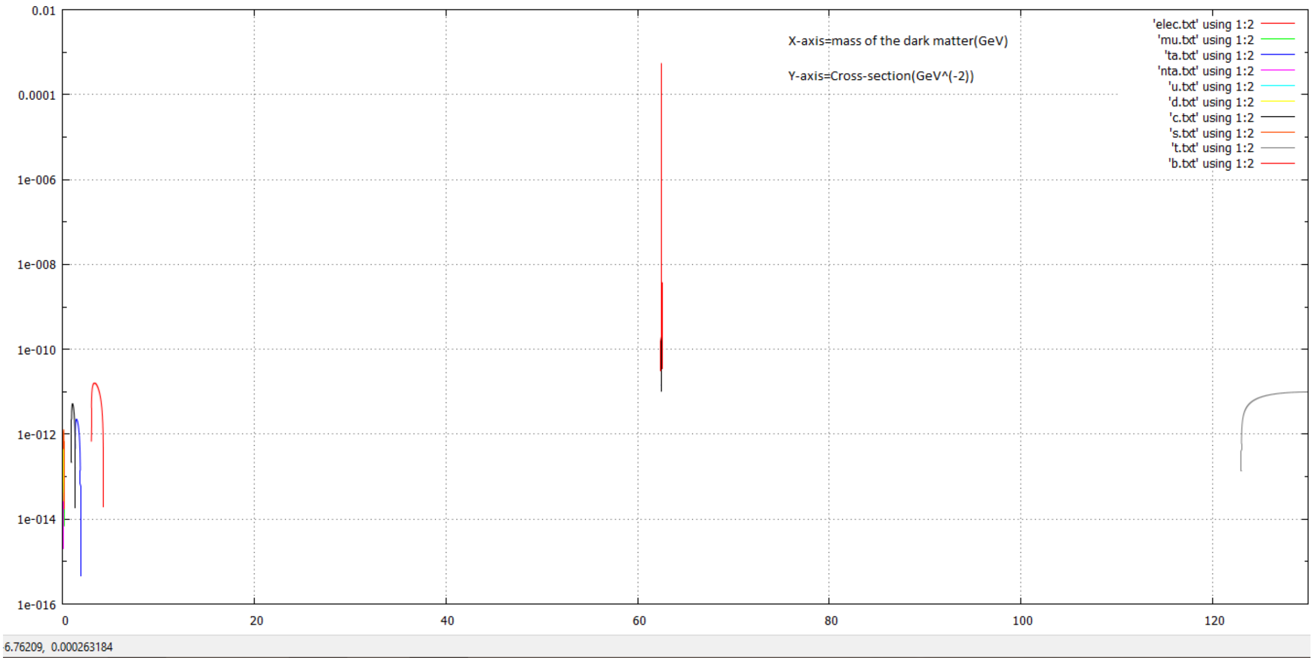


Figure 4.11: fermion contribution when x-range is taken 0 to 130 GeV

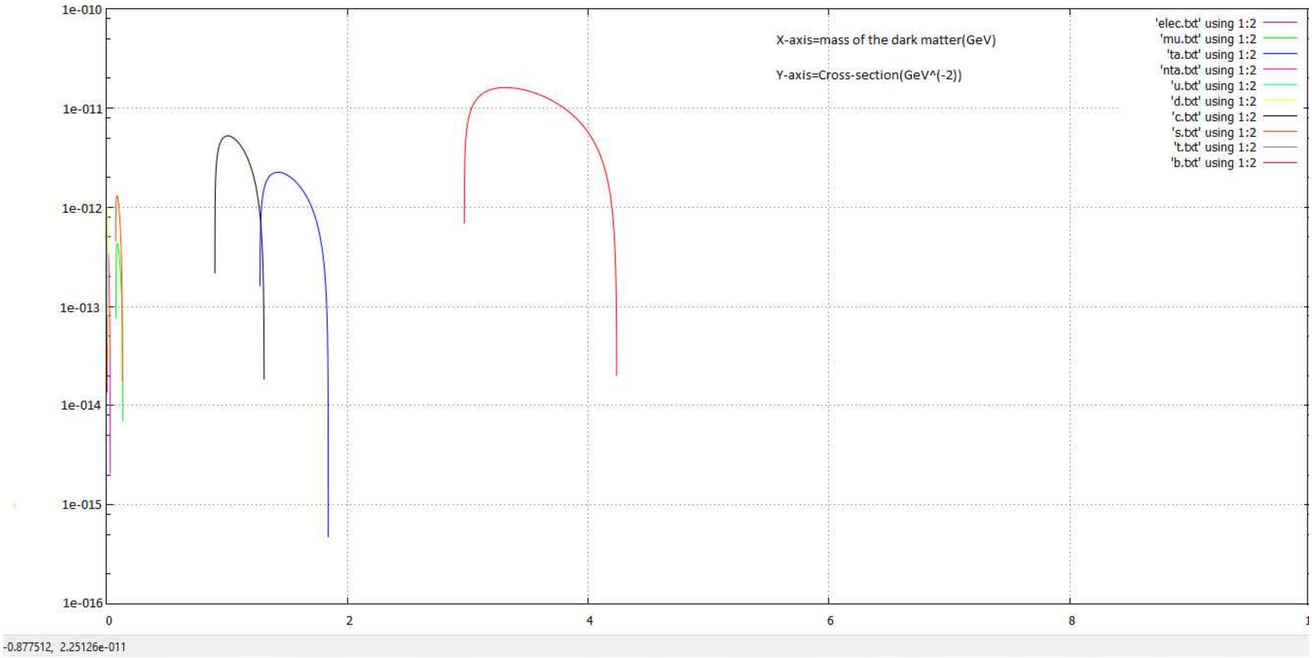


Figure 4.12: fermion contribution when x-range is taken 0 to 10 GeV

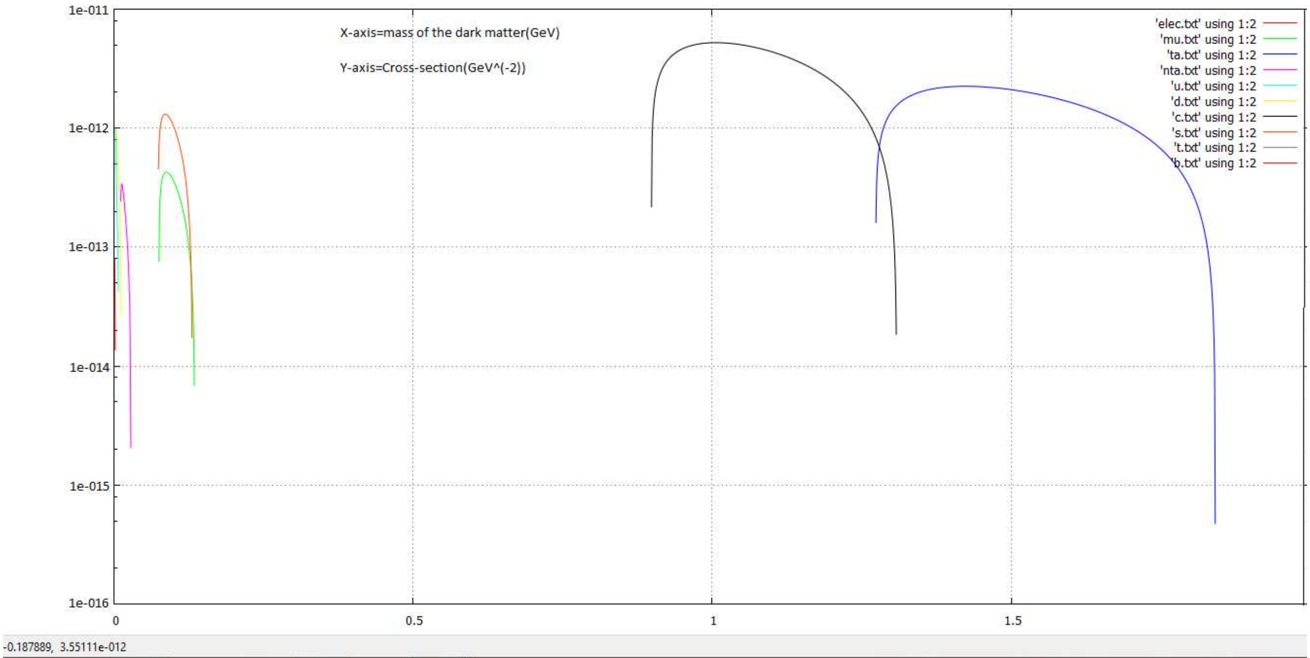


Figure 4.13: fermion contribution when x-range is taken 0 to 2 GeV

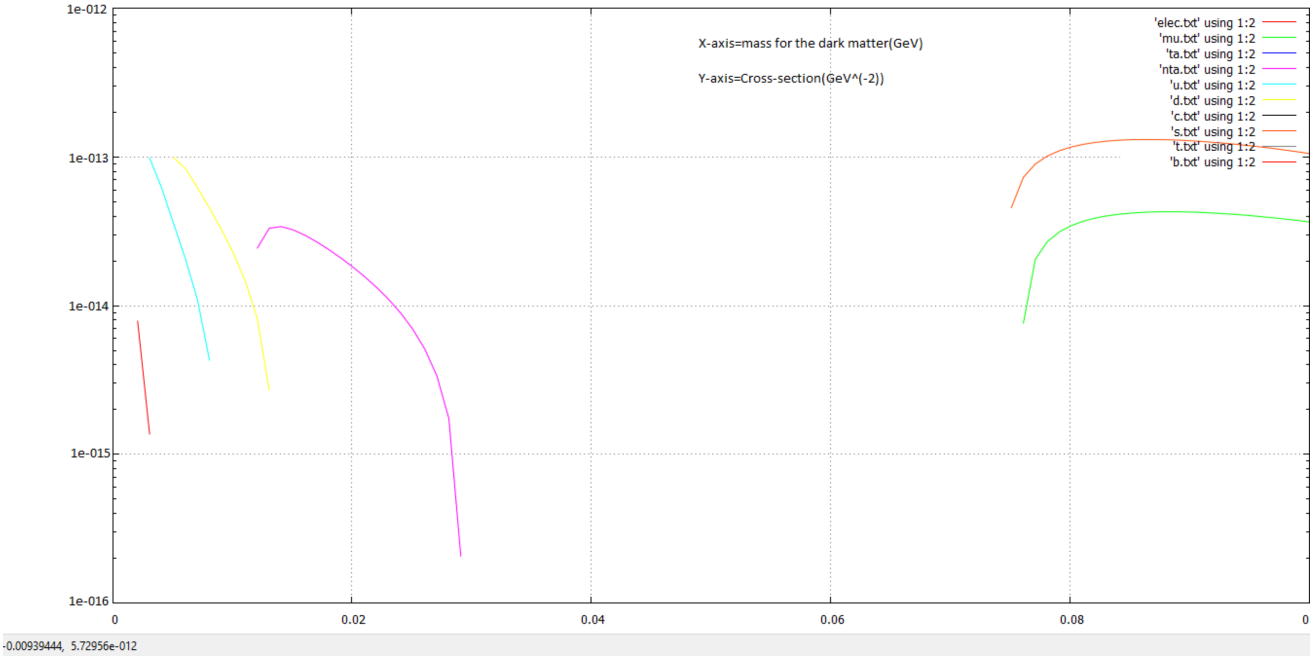


Figure 4.14: fermion contribution when x-range is taken 0 to 0.1 GeV

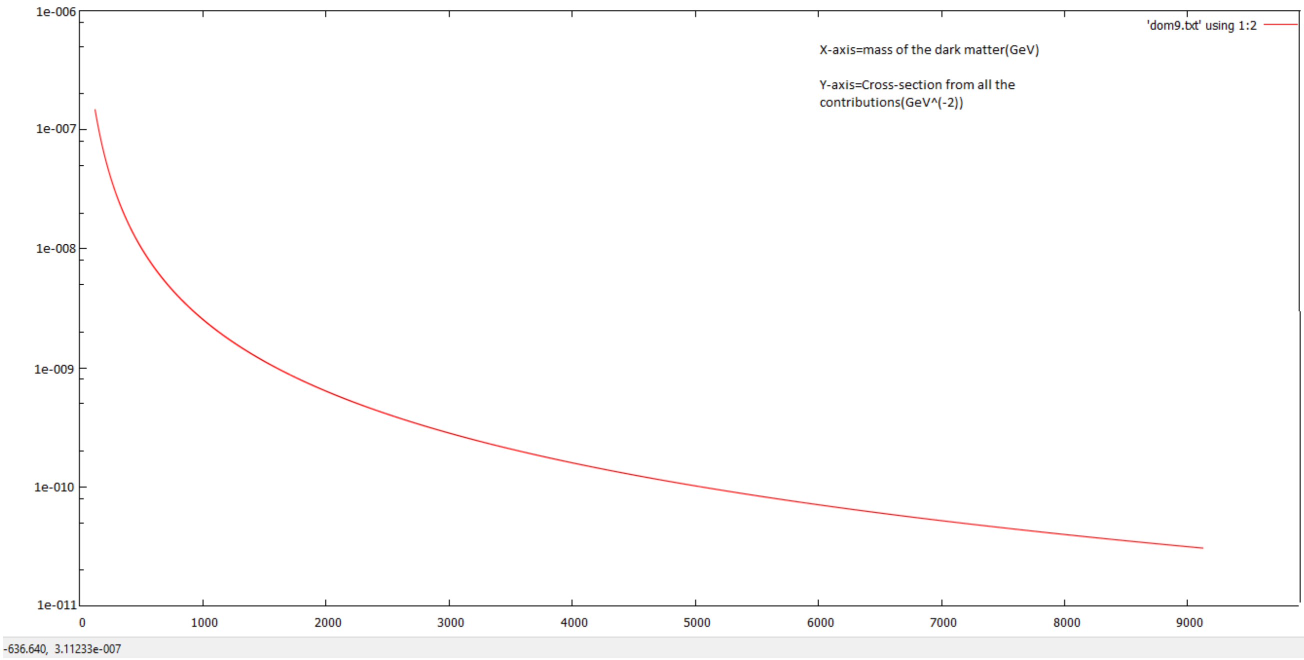


Figure 4.15: plot of total cross-section vs mass of dark matter

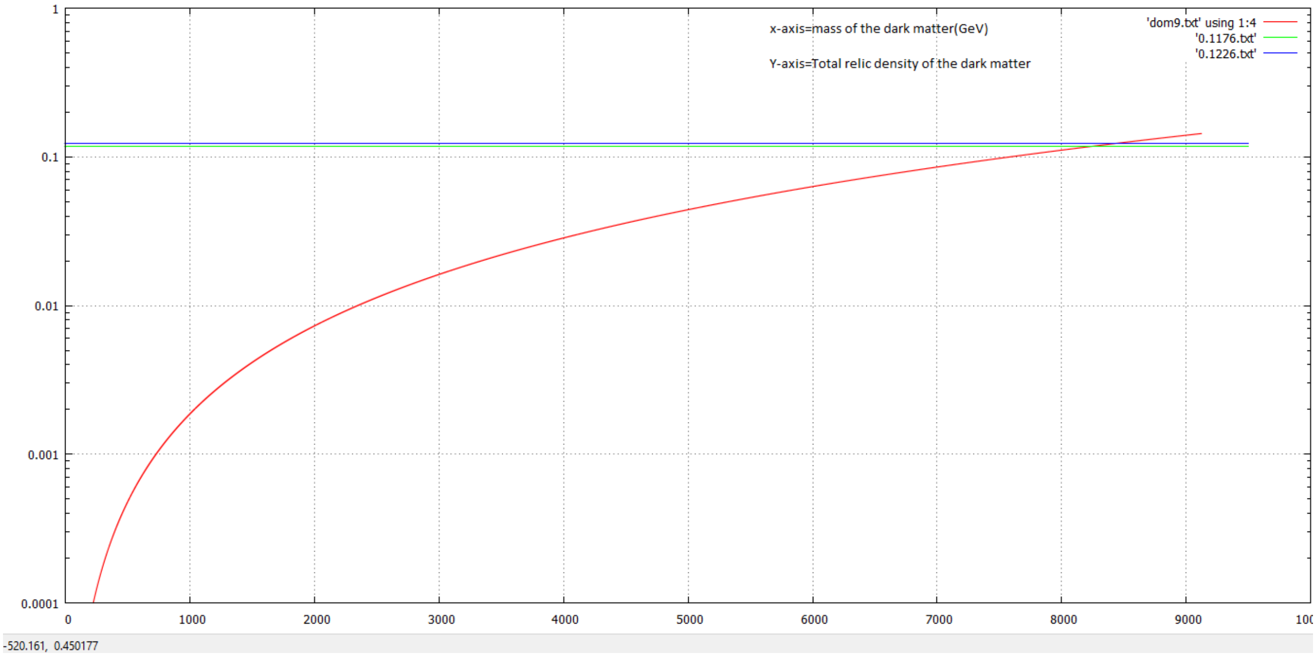


Figure 4.16: plot of total relic density vs mass of dark matter

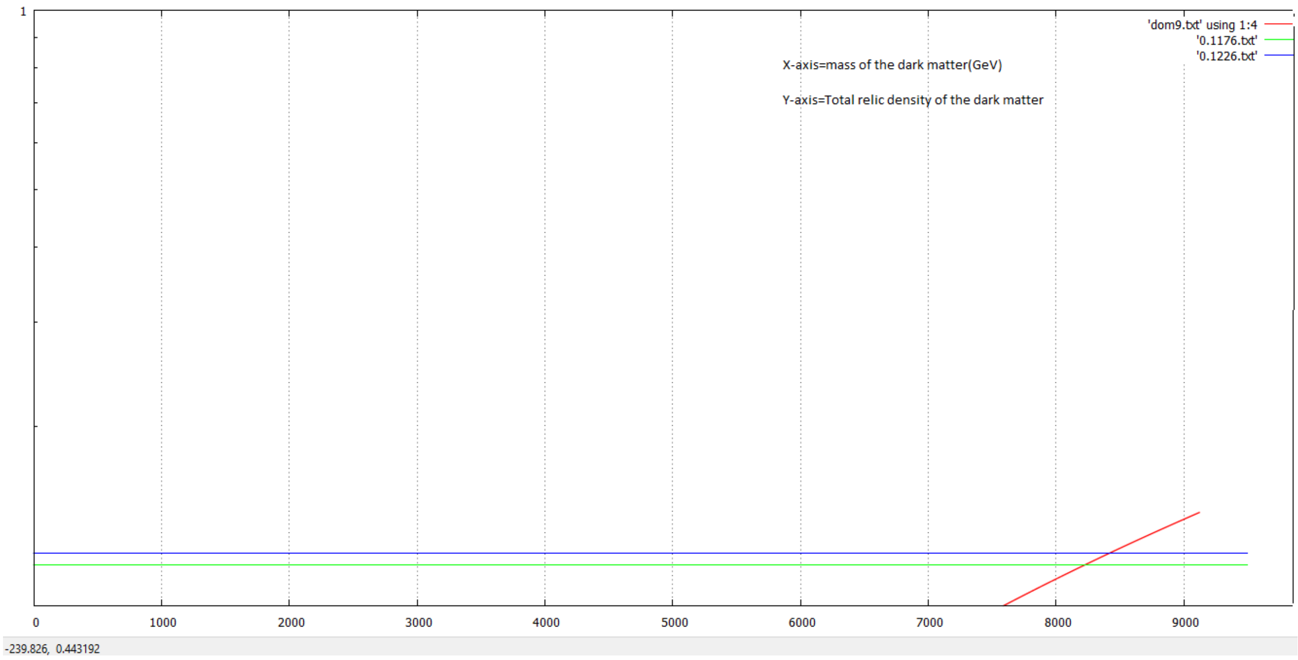


Figure 4.17: plot of total relic density vs mass of dark matter with the y range 0.1 to 1

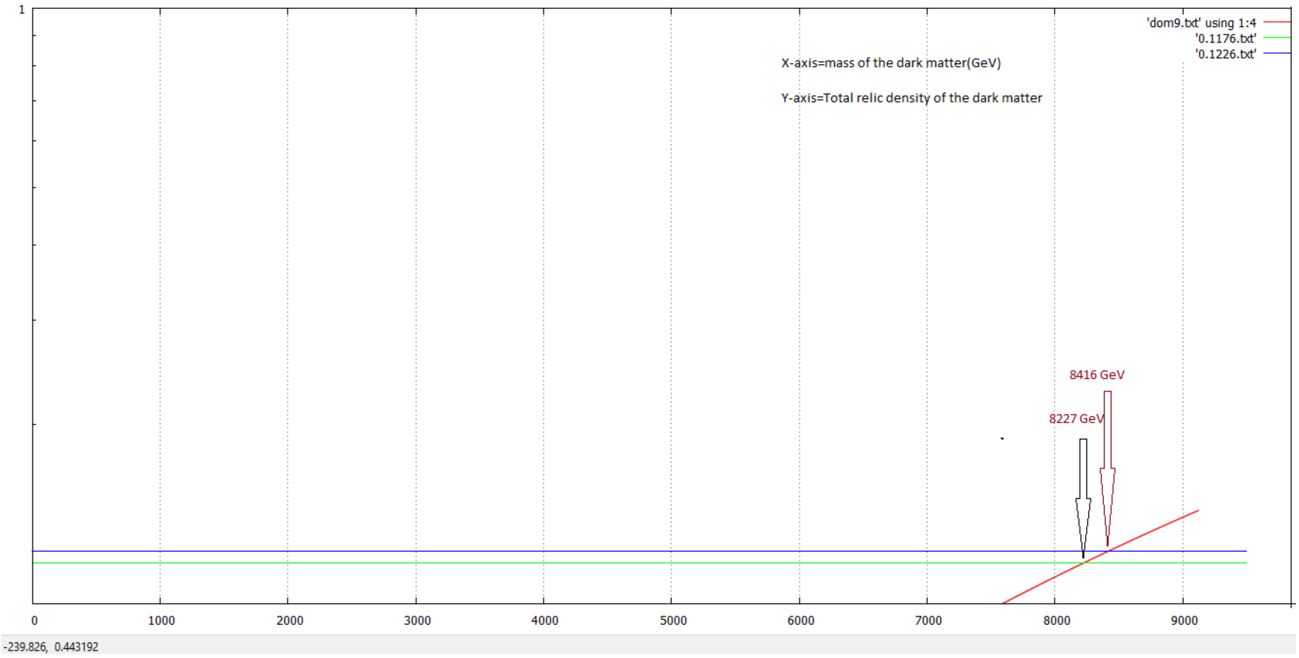


Figure 4.18: plot of total relic density vs mass of dark matter with the y range 0.1 to 1

Bibliography

[1] Footprint of Triplet Scalar Dark Matter in Direct,Indirect Search and Invisible Higgs Decay,[Seyed Yaser Ayazi and S.Mahdi Firouzabadi],arXiv:1501.06176v1[hep-ph]

[2] Standard Model: An Introduction[S.F.Novas],arXiv:hep-ph/0001283v1

[3] The Early Universe by Edward W. Kolb and Michael S. Turner

[4] Introduction to Cosmology by J.V.Narlikar

[5] Particle Dark Matter: Evidence, Candidates and Constraints[Gianfranco Bertone, Dan Hooper and Joseph Silk],arXiv:hep-ph/0404175v2

[6] TASI Lectures: Introduction to Cosmology[Mark Trodden and Sean M. Carroll],arXiv:astro-ph/0401547v1

[7] Dark Matter in Inert Triplet Models[Takeshi Araki,C. Q. Geng, and Keiko I. Nagao],arXiv:1102.4906v1 [hep-ph]

[8] A First Book of Quantum Field Theory(2nd Edition) by Amitava Lahiri and Palash B. Pal

[9] And all other Relativity,Relativistic quantum Mechanics,Quantum Field Theory books.