Suitability of FRIs based on Generalised Operators

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It is well known that a t-norm \(T\) and its residual implication \(I_T\), normally denoted as the residual pair \((T, I_T)\), play an important role in fuzzy inference systems, especially in Fuzzy Relational Inference (FRI) mechanisms. For instance, many desirable properties like the interpolativity, continuity, robustness and monotonicity of an FRI largely depend on the properties possessed by the residual pair \((T, I_T)\).

It is also well known that a more general class of a binary operations \(C : [0, 1]^2 \rightarrow [0, 1]\) than t-norms give rise to fuzzy implication \(I_C\) through the following residual operation:

\[
I_C(x, y) = \sup \{t \in [0, 1] | C(x, t) \leq y\}, \quad x, y \in [0, 1].
\]

For the exact conditions on \(C\) for \(I_C\) to be a fuzzy implication refer to [1]. Let us denote this class of conjunctions by \(\mathcal{C}\).

One can investigate the class \(\mathcal{C}\) along the following two approaches: On the one hand, one can discuss the additional conditions on \(C\) so that \(I_C\) satisfies some desirable properties of fuzzy implications like neutrality, ordering and exchange principle, etc. This has already been done quite extensively, see, for instance, [2], [4]. On the other hand, one can also investigate the additional conditions required on \(C\) that would enable a \((C, I_C)\) pair to be admitted in an FRI, see, for instance, [3]. In other words, if we call an FRI mechanism where a t-norm \(T\) is replaced by \(C\) and \(I_T\) replaced by \(I_C\) as a generalised FRI mechanism, then one can investigate the additional conditions required on \((C, I_C)\) for a generalised FRI to possess the above desirable properties. In this work, we follow the second line of investigation.

To this end, firstly, we discuss the solvability of the fuzzy relational equations (FREs) of the following form: For \(X, Y, Z\) being some universes of discourse and \(P, Q, R\) being some fuzzy relations on \(X \times Y, Y \times Z, X \times Z\) respectively,

\[
P \circ_C Q = R \text{ and } P \triangleleft_C Q = R
\]

where \(\circ_C = \sup -C\) and \(\triangleleft_C = \inf -I_C\) and determine the largest subset of \(\mathcal{C}\) that enjoys solvability of the above mentioned FREs.

Following this, we study the generalisations of two of the most important FRIs obtained from the above two types of FREs, viz., generalised Compositional Rule of Inference(CRI) and generalised Bandler-Kohout Subproduct(BKS) mechanisms and present some necessary and sufficient conditions on \(C\) that ensures some of the desirable properties like interpolativity and continuity in the considered setting.

We illustrate the above results with some specific family of conjunctions from \(\mathcal{C}\) which are not t-norms.


References


